SIMPLE STRESS, STRAIN

CHAPTER – 1

Q.1 Define Poisonous ratio 2006(w) (1-i), 2010(w)(1-a),2012(w)

Ans: If a body is stressed with in its elastic limit, then the lateral strain bears a constant ratio with the linear strain. This constant is known as poisonous ratio (Hooke's law)

Q2. Define Young's modulus of elasticity 2010(w), (1-b) 2006(w) (1-x)

Ans: It can be defined as the ratio of stress by strain of a stressed material

Elasticity = Stress/Strain i.e. $E = \sigma/\in$.

Q3. Srite relation between E, K and G 2010(w)(3), 2006(w)(1-ii)

Ans: Consider a wbe ABCD, A'B'C'D'.

Let the stress acting on faces = σ.

E = young's modulus of elasticity

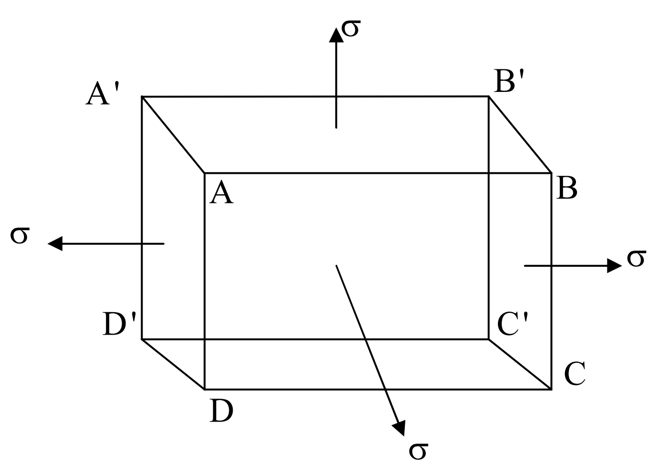
Consider deformation of face AB from ABCD

AB will suffer the following strains

(1) A tensile strain of σ/E.

We know that $E = 2C(1+1/m)$ ----------------- (i)

And Also $E = 3K(1-2/m)$ ---------------------- (ii)



Now

$$E = 2C\left(1+\frac{1}{m}\right) \qquad \Rightarrow \qquad E = 3K\left[\frac{C-E+2C}{C}\right]$$

$$\Rightarrow \frac{E}{2C} = 1+\frac{1}{m} \qquad \Rightarrow \qquad \frac{E}{3K} = \frac{C-E+2C}{C}$$

$$\Rightarrow \frac{1}{m} = \frac{E}{2C}-1 \qquad \Rightarrow \qquad \frac{E}{3K} = \frac{3C-E}{C}$$

$$\Rightarrow \frac{1}{m} = \frac{E-2C}{2C} \qquad \Rightarrow \qquad \frac{E}{3K} = 3-\frac{E}{C}$$

$$\Rightarrow m = \frac{2C}{E-2C} ----(iii) \qquad \Rightarrow \qquad \frac{E}{3K}+\frac{E}{C} = 3$$

Also

$$E = 3K\left(1-\frac{2}{m}\right) \qquad \Rightarrow \qquad \frac{EC+3KE}{3KC} = 3$$

$$\Rightarrow 3K\left(1-\frac{\frac{2}{2C}}{E-2C}\right) \qquad \Rightarrow \qquad EC+3KE = 9KC$$

$$\Rightarrow E = 3K\left(1-\frac{2(E-2C)}{2C}\right) \qquad \Rightarrow \qquad E = \frac{9KC}{3K+C}$$

$$\Rightarrow E = 3K\left(\frac{2C-2(E-2C}{2C}\right)$$

$$\Rightarrow E = 3K\left[\frac{C-(E-2C)}{C}\right]$$

This is the required Reculion between E,K and G

Q.4. Define strength of material 2007 (w)

Ans: A detailed study of analysis of forces with suitable protective measures for their safe working condition is known as strength of material.

Q.5 Define working stress 2007(w) (1-ii)

Ans: When a body is strained with in elastic limit then some resisting force or restoring force is offered by the body to deformation. This resisting force per unit area of the body is known as working stress.

Problem

A steel rod 25 mm in diameter and 2m long is subjected to an axial pull of 45 KN Find

(i) The intensity of stress

(ii) Strain

(iii) Elongation Take $E = 2 \times 10^5$ N/mm^2 2013(w), 1(c)

Given

$D = 25$ mm

$L = 2$ m $= 2000$ mm

$P = 45$ KN $= 45 \times 10^3$ N

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (25)^2 = 490.63\text{mm}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{45 \times 10^3}{490.63} = 91.7 \text{ N/mm}^2$$

$$\text{Strain, } \in = \frac{\text{stress}}{E} = \frac{91.7}{2 \times 10^5} = 0.00046$$

$$\text{Elongation, } \delta l = \frac{PL}{AE} = \frac{45 \times 10^3 \times 2000}{490.63 \times 2 \times 10^5}$$

$$= 0.92\text{mm}$$

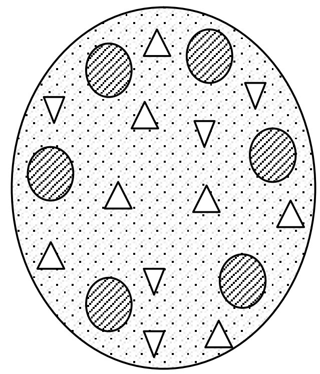
Problem:

A reinforced concrete circular column 50000 mm^2 cross sectional area carries six rein forcing bars whose total area is 500 mm^2. Find the safe load the column can carry if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.

2013(w),3(c)

Area of column (A) = 5000 mm^2

Area of 6 steel bars (A_s) = 500 mm^2



Area of concrete, $A_c = A - A_s$ = 50,000 − 500

= 49500 mm^2

Stress in concrete, σ_c = 3.5 MPa = 3.5 N/mm^2

Let σ_S = stress in steel

Modular ratio = $E_S/E_C = 18$

$\sigma_S/\sigma_C = E_S/E_C = 18 \Rightarrow \sigma_S = 18\ \sigma_C = 18 \times 3.5 = 63\ N/mm^2$

$P = \sigma_C\ ,\ A_c + \sigma_s.A_S$

= (3.5 × 49,5000) + (63 × 500) = 173250 + 31500 = 204750 N = 204.75 KN.

Problem :

A rod of steel is 20 m long at a temperature of 20°C. Find the free expansion of rod when temperature is raised by 65°C. also find the temperature stress when expansion of rod is prevented Take $\alpha = 12\ 10^{-6}/^{o}C$ and $E = 2\ 10^{5}\ N/mm^2$ 2013(w), 4(c)

Given :

$L = 20 \text{ m} = 20,000 \text{ mm}$

Rise in temperature, $t = 65^o - 20^o = 45^oC$

$\alpha = 12 \times 10^{-6}/^oC \quad E = 2 \times 10^5 \text{ N/mm}^2$

Expansion of rod, $l = l\ \alpha\ t$

$= 20,000 \times 12 \times 10^{-6} \times 45 = 10.8 \text{ mm}$

Temperature stress, $= \alpha\ t\ E = 12 \times 10^{-6} \times 45 \times 2 \times 10^5 = 108 \text{ N/mm}^2$

Q. State Hooke's law 2014(w)

Ans: When material is loaded within elastic limit, stress is proportional to strain.

Mathematically stress α strain.

i.e. $\frac{\text{stress}}{\text{strain}} = E = \text{constant}$

Where E = young's modulus of elasticity. Define stress and strain 2014(w)

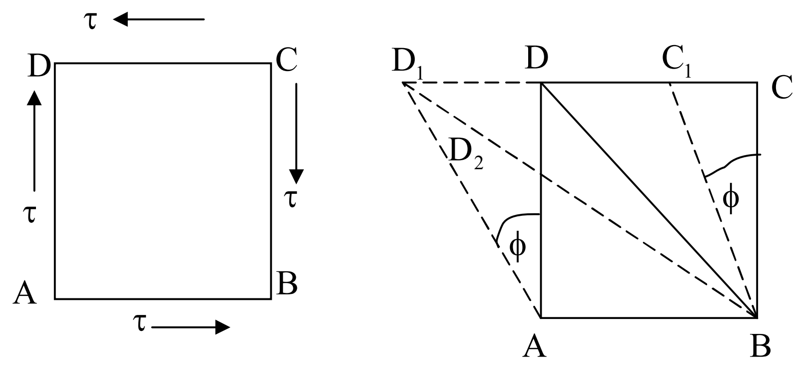
Stress – The restoring force per unit area is known as stress

$$\text{Stress}(\sigma)\frac{\text{Force}}{\text{Area}} = \frac{P}{A}$$

Strain- The deformation per unit length is known as strain.

Strain , $e = \delta l/L$

Q. State relation between modulus of elasticity and modulus of rigidity (c).2014(w)



Consider a cube of length ‘l’ subjected to a shear stress τ as shown in figure. A little consideration will show that due to these stresses the cube is subjected to some distortion such that the diagonal BD will be elongated and diagonal Ac will be shortened. Let this shear stress (τ) cause shear stress as shown. We see that diagonal BD is distorted to BD.

$$\text{Strain of BD} = \frac{BD_1 - BD}{BD} = \frac{D_1D_2}{BD} = \frac{DD_1 COS45}{AD\sqrt{2}}$$

$$= \frac{DD_1}{2AD} = \frac{\phi}{2}$$

We see that the linear strain of diagonals BD is half of shear strain and is tensile in nature. Similarly it can be proved that the linear strain of diagonal AC is also equal to half of shear strain but is compressive in nature, Now this linear strain of diagonal $BD = \frac{\phi}{2} = \frac{\tau}{2C} ------(1)$

Where τ = shear stress

C= Modulus of rigidity

Let us now consider this shear stress (τ) acting on the sides AB, CD, CB and AD. Now the effect of this stress is to cause tensile stress an diagonal BD and compressive stress on diagonal AC.

Therefore tensile stress on diagonal BD due to tensile stress on diagonal

$$BD = \frac{\tau}{E} ------(2)$$

Tensile strain on diagonal Bd due to compressive stress on diagonal

$$AC = \frac{1}{m} \times \frac{\tau}{E} ------(3)$$

The combined effect of above two stress on diagonal

$$BD = \frac{\tau}{E} + \frac{1}{m} \times \frac{\tau}{E} = \frac{\tau}{E}\left(1 + \frac{1}{m}\right) = \frac{\tau}{E}\left(\frac{m+1}{m}\right) ------(4)$$

Now equating equations (1) and (2)

$$\frac{\tau}{2C} + \frac{\tau}{E}\left(\frac{m+1}{m}\right) \quad \text{or} \quad C = \frac{mE}{2(m+1)}$$

$$= \frac{2.86 \times 318480}{3(2.86-2)} = 353043.7\,N/mm^2$$

Problem:

A composite bar is made up of brass rod of 25 mm diametrer enclosed in a steel tuber of 40 mm external and 35 mm internal diameter. The ends of rod and tube are securely fixed. Find stresses developed is rod and steel tube when the composite bar is subjected to an axial pull of 45 KN. Take E for brass as 80 GPa and E for steel as 200 GPa 2012(w)(2c)

Given:

Diameter of brass road d_b = 25 mm

Area of brass rod, $A_b = \pi/4 \times d_b^2 = \pi/4 \times (25)^2 = 490.63\ mm^2$

Area of steel tube, $A_s = \pi/4(40^2 - 35^2) = 294.38\ mm^2$

$P = 45\ KN = 45 \times 10^3\ N$

Let σ_b = stress is brass

σ_S = stress is steel

$E_b = 80$ GPa $= 80 \times 10^3$ N/mm^2

$E_s = 200$ GPa $= 200 \times 10^3$ N/mm^2

$$\frac{\sigma_s}{\sigma_b} = \frac{E_s}{E_b} = \frac{200 \times 10^3}{80 \times 10^3} = 2.5 \qquad \Rightarrow \sigma_s = 2.5\sigma_b$$

$$P = \sigma_s.A_s + \sigma_b.A_b \quad \Rightarrow 45 \times 10^3 = 2.5\sigma_b \times 294.38 + \sigma_b \times 490.63$$

$$\Rightarrow 45 \times 10^3 = \sigma_b\left[(2.5 \times 294.38) + 490.63\right]$$

$$\Rightarrow 45 \times 10^3 = \sigma_b(735.95 + 490.63) \qquad \Rightarrow 1226.6\sigma_b = 45 \times 10^3$$

$$\Rightarrow \sigma_b = \frac{45 \times 10^3}{1226.6} = 36.7 \text{ N/mm}^2$$

$$\sigma_s = 2.5\sigma_b = 2.5 \times 3.67 = 91.75 \text{ N/mm}^2$$

Problem

A bar of 20 mm diameter is subjected to a pull of 50 KN. The measured extension over a gauge length of 20 cm is found to be 0.1 mm and change in diameter is 0.0035 mm Evaluate the poisonous ratio, e and is:

2015(w), (1-c)

Diameter of bar, d = 20 mm

Area of bar, $A = \pi/4 \times d^2 = \pi/4 \times (20)^2 = 314$ mm^2

Length of bar, L = 20 cm = 200 mm

Extension of bar, δL=0.1 mm $\qquad$ $P = 50$ KN $= 50 \times 10^3$ N

Change in diameter, d = 0.0035 mm

$$\text{Linear strain}, e = \frac{\delta l}{l} = \frac{0.1}{200} 0.0005$$

$$\text{Lateral strain}, = \frac{\delta d}{d} = \frac{0.0035}{20} = 0.000175$$

$$\text{Poisonou's ratio}, \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{0.000175}{0.0005} = 0.35$$

$$\text{or} \qquad m = \frac{1}{0.35} = 2.86$$

$$\text{Stress}, \delta = \frac{P}{A} = \frac{50 \times 10^3}{314} = 159.24\ N/mm^2$$

$$\text{strain}, e = \frac{\delta L}{L} = \frac{0.1}{200} = 0.0005$$

$$\text{Young's modulus}, E = \frac{\text{Stress}}{\text{Strain}} = \frac{159.24}{0.0005} = 318480\ N/mm^2$$

$$\text{Bulk modulus}, K = \frac{mE}{3(m-2)}$$

Problem :

A tensile load of 60 KN applied axial on a cylindrical bar of diameter 10 cm. What is the tensile stress on a section perpendicular to the axis of bar 2010(w),2014(w) 1(b)

Load, P = 60 KN = 60 × 10 N

Diameter, d = 10 cm = 0.1 m

Area, A = $\pi/4 \times d^2 = \pi/4 \times (0.1)^2 = 0.00785\ m^2 = 78550\ mm^2$

$$\text{Stress}, \quad = \frac{P}{A} = \frac{60 \times 10^3}{7850} N/mm^2 = 7.64\ N/mm^2$$

Problem:

A material has a Young’s modulus 1.3 × 10^5 N/mm^2 and poisonous ratio of 0.3. Calculate rigidity modulus and bulk modulus 2014(w) 2(b)

Young's modulus, $E = 1.3 \times 10^5\ N/mm^2$

Poisonous ratio, $1/m = 0.3$ or $m = 1/0.3 = 3.33$

$$\text{Bulk modulus, } K = \frac{mE}{3(m-2)}$$

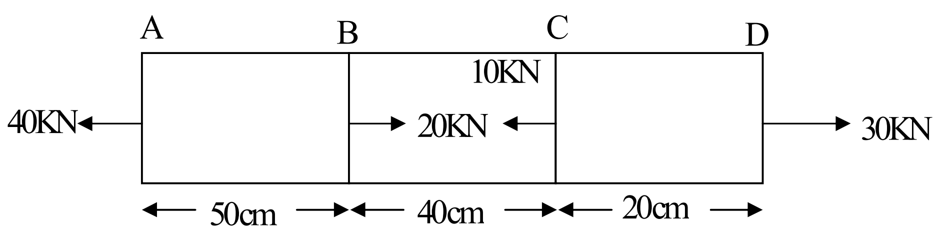
$$= \frac{3.33 \times 1.3 \times 10^5}{3(3.33-2)} = \frac{3.33 \times 1.3 \times 10^5}{3 \times 1.33} = 1.08 \times 10^5\ N/mm^2$$

$$\text{Modulus of rigidity, } C = \frac{mE}{2(m+1)} = \frac{3.33 \times 1.3 \times 10^5}{2(3.33+1)}$$

$$= \frac{3.33 \times 1.3 \times 10^5}{2 \times 4.33} = 0.5 \times 10^5\ N/mm^2$$

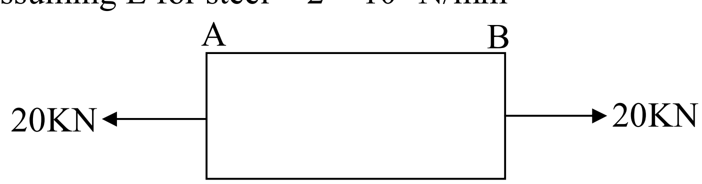
Problem :

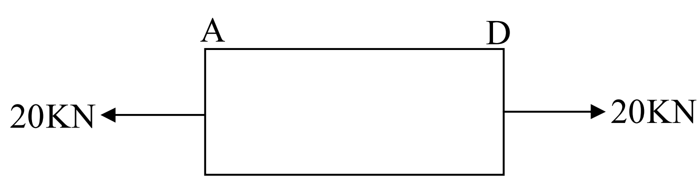
A steel bar 25 mm diameter is loaded as shown in figure. Determine stresses in each part of the total elongation 2014(w)

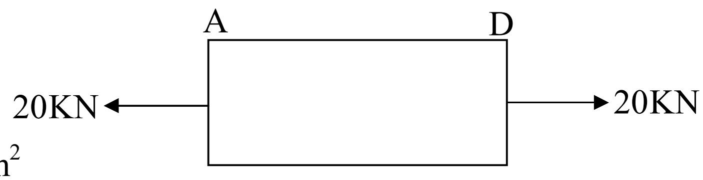


Let dl = total elongation Assuming E for steel = $2 \times 10^5\ N/mm^2$

Total elongation



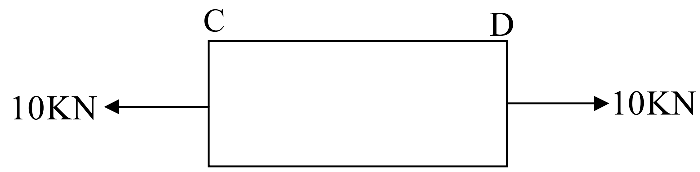




Area of steel bar

$A = \pi/4 \times (25)^2 = 490.625\ mm^2$

Total elongation



$$(\delta l) = \frac{P_1L_1}{AE} + \frac{P_2L_2}{AE} + \frac{P_3L_3}{AE}$$

$$= \frac{1}{AE}(P_1L_1 + P_2L_2 + P_3L_3)$$

$$= \frac{1}{490.625 \times 2 \times 10^5}\left[20 \times 10^3 \times 500 + 20 \times 10^3 \times 1100 + 10 \times 10^3 \times 200\right]$$

$$= \frac{1}{490.625 \times 2 \times 10^5} \times 10^3\left[20 \times 500 + 20 \times 1100 + 10 \times 200\right]$$

$$= \frac{1}{490.625 \times 2 \times 100}\left[10,000 + 22,000 + 20,000\right] = 0.35mm$$

Problem:

A 15 cm dia steel rod passes centrally through a copper tube 50 mm external dia and 40 mm internal dia. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of rod. If the temperature of assembly is raised by 60°C, calculate the stresses raised by 60°C, Calculate the stresses developed in steel and copper. Take E for steel and copper as 210 kw/mm² and 110 KN/mm² respectively. Also α for steel and copper as 12×10^{-6}/°C and 17.5×19^{-6}/°C respectively 2014(w), 7(c)

Given

Diameter of steel rod $d_s = 15$ cm

Area of steel rod, $A_s = \pi/4 \times d_s^2 = \pi/4 \times (15)^2\ cm^2 = 176.63\ mm^2$

Area of copper tube, $A_c = \pi/4\ (50^2 - 40)^2 = 706.5\ mm^2$

$t = 60^o c$

Tension in steel = Compression in copper

$\sigma_S . A_S = \sigma_c . A_C$

$$\Rightarrow \frac{\sigma_s}{\sigma_c} = \frac{A_c}{A_s} = \frac{706.5}{176.63} = 4 \Rightarrow \sigma_s = 4\sigma_c$$

$$E_s + E_c = t(\alpha_c - \alpha_s)$$

$$\Rightarrow \frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c} = 60\left[17.5\times10^{-6} - 12\times10^{-6}\right]$$

$$\Rightarrow \frac{4\sigma_c}{210\times10^3} + \frac{\sigma_c}{110\times10^3} = 60\times5.5\times10^{-6}$$

$$\Rightarrow \frac{\sigma_c}{10^3}\left[\frac{4}{210} + \frac{1}{110}\right] = 60\times5.5\times10^{-6}$$

$$\Rightarrow \frac{\sigma_c}{10^3}\left[0.019 + 0.00009\right] = 60\times5.5\times10^{-6}$$

$$\Rightarrow \frac{\sigma_c}{10^3}\times0.01 = 60\times5.5\times10^{-6} \Rightarrow \sigma_c = 33N/mm^2$$

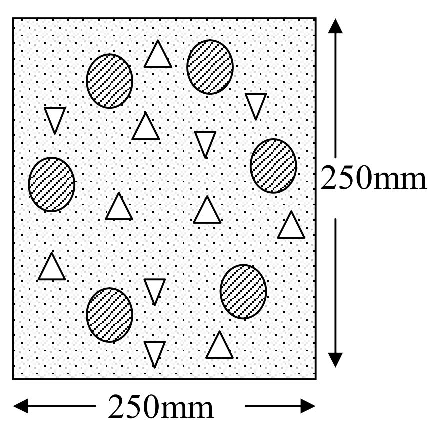
$$\sigma_s = 4\sigma_c = 4\times33 = 132\,N/mm^2$$

Problem:

A reinforced short concrete column 250 mm × 250 mm section is reinforced with 8 number of steel bars. The total area of steel bars is 2500 mm^2. The column is carrying a load of 390 KN. Find the stresses in concrete and steel. Assume E_s = 15 E_c 2015(w), 3(c)

Area of concrete column (A) = 250 250 = 62500 mm^2

Area of 8 steel bars (A_s) = 2500 mm^2



Area of concrete,

$A_c = A - A_s$ = 62500 − 2500 = 60,000 mm^2

E_s = 15 E_c

Let σ_s = stress in steel $\Rightarrow E_s$ = 15

σ_c = stress in concrete E_c

P = 390 KN = 390 × 10^3 N

$\sigma_s/\sigma_c = Es/E_c = 15 \Rightarrow \sigma_s = 16\ \sigma_c$

$P = \sigma_c + \sigma_s . A_s \Rightarrow 390 \times 10^3 = \sigma_c \times 60,000 + 15\ \sigma_c \times 2500$

$= \sigma_c(60,000 + 37,500) = 97500\ \sigma_c$

$$\Rightarrow \sigma_c = \frac{390 \times 10^3}{97500} 4N/mm^2$$

$$\sigma_s = 15 \times \sigma_c = 15 \times 4 = 60N/mm^2$$

CHAPTER:2

Q.1 **Define temperature stress. 2005(w), 1(j), 2012(w), 2(a) 2014(w)**

Ans: When ever a body is subjects to a change in temperature it undergoes expansion or contraction. But if the deformation of the body is prevented, then the stress which will induced in the body is known as temp. stress.

Q2. Define hoop stress and longitudinal stress.

2012(w), 3(a), 2005(w), 1(c) 2013(w), 5(a), 2014(w)

Ans: Hoop stress: The stress which acts tangentially along the circumference of the shell, this is known as circumferential stress is σ_c

Longitudinal stress: The stress which acts parallel to the longitudinal axis of the shell is known as longitudinal stress σ_c.

Q3. Derive an expression for hoop stress and longitudinal stress for a thin cylinder subjected to an internal pressure 'P' 2012(w) 3(b), 2005(w),2(d), 2006,(2c), 2013, (5b), 2014(w),2015(w), (2b)

Ans: Let l = length of the shell.

P = Intensity of internal pressure

σ_c = circumferential stress.

d = diameter of the circular shell.

t = thickness.

Total pressure along $x - x'$ = Intensity of pressure × Area

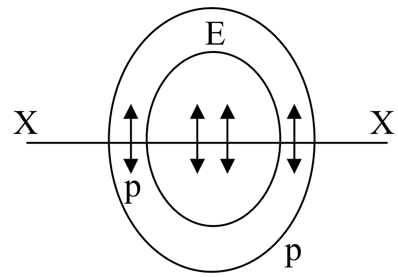
$= P \times d \times l$

Resisting section = 2t l

Circumferential stress $\sigma_c = \frac{\text{Total pressure}}{\text{Resisting section}}$

$$= \sigma_c = \frac{P \times d \times l}{2tl}$$

$$\sigma_c = \frac{Pd}{2t}$$



Longitudinal stress :-

Now total pressure acting along y - y'

= Intensity of pressure × Area.

$= P\,\pi/4\,d^2$

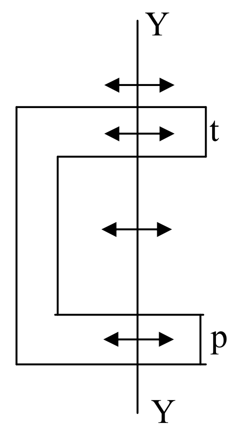
Resisting section = π d × t

Longitudinal stress

$$\sigma_L \frac{\text{Total pressure along } yy'}{\text{Resisting section.}}$$

$$= \frac{P \times \pi d^2}{4\pi dt} = \frac{pd}{4t}$$

$$= \sigma_L = \frac{pd}{4t}$$



Q. Find expression for temperature stress for a rise in temperature of t°C. when the ends do not yield. Take α co-efficient of expression 'l' as the original length 2014(w), 2015(w) 1(b)

Ans: Consider a body subjected to an increase in temperature.

Let l = original length of body

T = Increase of temperature

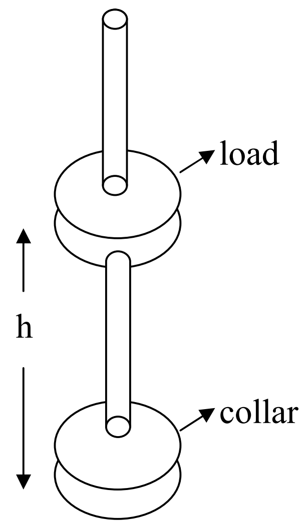
α = Co-efficient of linear expansion

Increase in length due to increase of temperature, $\delta l = l\,\alpha\,t$

When the ends do not yield

Strain, $e = \frac{\delta l}{l} = \frac{l \alpha t}{l} = \alpha t$

Find out stress due to impact loading



Consider a bar subjected to a load applied with impact as shown in figure.

Let p = load applied with impact

A = cross sectional area of bar

E = Modulus of elasticity of bar material

δl = Deformation of bar

σ = stress induced by the application of this load with impact

h = height through which load will fall.

Work done = load × distance = p(h + δl) and energy stored, $u = \frac{\sigma^2}{2E} \times Al$

Since energy = workdone

$$\therefore \frac{\sigma^2}{2E} \times Al = p(h + \delta l) = p(h + \frac{\delta}{E} l)$$

$$\therefore \frac{\sigma^2}{2E} \times Al = ph + \frac{p\sigma l}{E}$$

$$\therefore \frac{\sigma^2}{2E} \times Al = \frac{p\sigma l}{E} - ph = 0$$

Multiplying both sides by E/Al

$$\therefore \frac{\sigma^2}{2} - \sigma\left(\frac{p}{A}\right) - \frac{pEh}{Al} = 0$$

This is a quadratic equation

$$\therefore \sigma = \frac{p}{A} \pm \sqrt{\left(\frac{p}{A}\right)^2 + 4 \times \frac{1}{2} \times \frac{pEh}{Al}}$$

$$= \frac{p}{A}\left[1 \pm \sqrt{1 + \frac{2AEh}{pl}}\right]$$

Q. Define strain energy and resistance 2015(w), 2(a)

Strain energy : The amount of energy stored in a body when strained within elastic limit is known as strain energy.

Strain energy = work done

Resistance: The strain energy stored in a body when strained within elastic limit is known as resistance.

Problem:

A cylindrical shell 2.5 m long and closed at the ends has an internal diameter of 1.25 m and wall thickness of 20 mm. Calculate the change in dimensions when subjected to an internal pressure of 1.5 MPa. Take E = 200 GPa and 1/m = 0.3 2014(w), 2(c)

Given

L = 2.5 m = 2500 mm.

D = 1.25 m = 1250 mm

T = 20 mm

P = 1.5 Mpa = 1.5 N/mm^2

E = 200 GPa = 200 × 10^3 N/mm^2

1/m = 0.3

Circumferential stress, $\sigma_c = pd/2t$ N/mm^2

Longitudinal stress $\sigma_l = pd/4t = N/mm^2$

$$\text{Change in diameter}, \delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times (1250)^2}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right) = 0.24\text{mm}$$

$$\text{Change in length}, \delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{1.5 \times 1250 \times 2500}{2 \times 20 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.117\text{mm}$$

Problem:

A cylindrical shell 4m long has 1 m internal diameter and 20 mm metal thickness. Calculate the circumferential and longitudinal stress. If the shell is subjected to an internal pressure of 2Mpa. Calculate change in dimension of shell Take E = 200 GPa and poisonous ratio = 0.3

2014(w), 3(c)

l = 4m = 4000 mm

d = 1m = 1000 mm

t = 20 mm

p = 2 MPa = 2 N/mm^2

E = 200 GPa = 200 × 10^3 N/mm^2

1/m = 0.3

Circumferential stress, $\sigma_c = \frac{pd}{2t} = \frac{2 \times 1000}{2 \times 20} = 50\, N/mm^2$

Longitudinal stress, $\sigma_l = \frac{pd}{4t} = \frac{2 \times 1000}{4 \times 20} = 25\, N/mm^2$

Change in diameter, $\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{2 \times (1000)^2}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right)$$

$$= 0.2125 \text{ mm}$$

Change in length, $\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$

$$= \frac{2 \times 1000 \times 4000}{2 \times 20 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right)$$

$$= 0.2 \text{ mm}$$

Change in volume = ?

$$\text{Hoop strain } \in_c = \frac{pd}{2tE}\left(1-\frac{1}{2m}\right) = \frac{2\times1000}{2\times20\times200\times10^3}\left(1-\frac{1}{2}\times0.3\right)$$

$$= 0.00021$$

$$\text{Longitudinal strain, } \in_l = \frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{2m}\right)$$

$$= \frac{2\times1000}{2\times20\times200\times10^3}\left(\frac{1}{2}-0.3\right)$$

$$= 0.00005$$

$$\text{Volume of shell, } V = \frac{\pi}{4}\times d^2\times l = \frac{\pi}{4}(1000)^2\times4000$$

$$= 3.25\times10\text{mm}$$

$$\frac{\delta V}{V} = 2\in_c + \in_l \quad \text{or} \quad \delta V = v\left[2\in_c + \in_l\right] = 3.25\times10^{-12}\left[2\times0.00021+0.0005\right]$$

$$= 3\times10^{-15}\text{mm}^3$$

Problem:

A cylindrical vessel closed with plane ends is made of 4 mm thick steel plate. The diameter and length are 250 mm and 750 mm respectively when same is subjected to an internal pressure of 300 N/mm^2. Calculate the following

(i) Longitudinal and hoop stress

(ii) Changes in diameter, length and volume

Assume e = 200 G N/m^2

Poisonous ratio = 0.3 2015(w), 4(c)

Given:

t = 4 mm

d = 250 mm

l = 750 mm

p = 300 N/cm^2 = 3 N/mm^2

E = 200 G N/m^2 = 200 × 10^3 N/mm^2, 1/m = 0.3

Circumferential stress, $\sigma_c = \frac{pd}{2t}$

$$= \frac{3 \times 250}{2 \times 4} = 93.75 \ \ N/mm^2$$

Longitudinal stress, $\sigma_l = \frac{pd}{4t}$

$$= \frac{3 \times 250}{4 \times 4} = 46.88 \ \ N/mm^2$$

Change in diameter, $\sigma d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{3 \times 250 \times 750}{2 \times 4 \times 200 \times 10^3} = \left(\frac{1}{2} - 0.3\right) = 0.007mm$$

Problem:

A cylindrical vessel 2m to 500 mm in diameter with 10 mm plate is subjected to an internal pressure of 3 MPa. Calculate change volume of vessel. Take E = 200 GPa, Poisonous ratio = 0.3 for the vessel material.

2013(w), 5(c)

Given :

$l = 2m = 2000$ mm

$d = 500$ mm

$p = 3$ MPa $= 3N/mm^2$

$E = 200$ GPa $= 200 \times 10^3 \ N/mm^2$

$1/m = 0.3$

$t = 10$ mm

Volume of cylinder, $V = \frac{\pi}{4} \times d^2 \times L$

$$= \frac{\pi}{4} \times (500)^2 \times 2000 = 392500000 mm^3$$

Hoop strain, $\in_c = \frac{pd}{2tE} - \frac{1}{m} \times \frac{pd}{4tE} = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{3 \times 500}{2 \times 10 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right) = 0.000319$$

Longitudinal strain, $\in_l = \frac{pd}{4tE} - \frac{1}{m} \times \frac{pd}{2tE}$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) = \frac{3 \times 500}{2 \times 10 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.000075$$

Volumetric strain, $\frac{\delta V}{v} = 2 \in_l + \in_c$

$$\Rightarrow \delta V = V(2 \in_l + \in_c)$$

$$= 392500000(2 \times 0.000075 + 0.000319)$$

$$= 184475 mm^3$$

Change in volume $184475 mm^3$

Problem:

A cylindrical shell 3 m long has 1n internal diameter and 15 mm metal thickness. Calculate the circumferential and longitudinal stresses if the shell is subjected to an internal pressure of 1.5 MPa. Also calculate change in dimension of shell. Take E = 200 GPa and poisonous ratio = 0.3 2012(w), 3(c)

Givcn :

l = 3 m = 3000 mm

d = 1m, = 1000 mm

t = 15 mm

$p = 1.5$ MPa = 1.5 N/mm^2

$E = 200$ GPa = 200×10^3 N/mm^2

$1/m = 0.3$

$$\text{circumferential stress}, \sigma_c = \frac{pd}{2t} = \frac{1.5 \times 1000}{2 \times 15} = 50\,N/mm^2$$

$$\text{longitudinal stress}, \sigma_l = \frac{pd}{4t} = \frac{1.5 \times 1000}{4 \times 15} = 25\,N/mm^2$$

$$\text{change in diameter}, \delta_d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times (1000)^2}{2 \times 15 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right)$$

$$= 0.21\text{ mm}$$

$$\text{Change in length } \delta_l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{1.5 \times 1000 \times 3000}{2 \times 15 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.15\text{mm}$$

Problem :

A cylindrical shell 2.5 m way and closed at the ends has an internal diameter of 1.25 m and wall thickness of 20 mm. Calculate the change in dimension when subjected to an internal pressure is 1.5 MPa. Take E = 200 GPa and 1/m = 0.3 2014(w), 2(c)

Given:

$l = 2.5$ m = 2500 mm

$d = 1.25$ m = 1250 mm

$t = 20$ mm

$p = 1.5$ MPa = 1.5 N/mm^2

$E = 200$ GPa $= 200 \times 10^3$ N/mm^2

$1/m = 0.3$

Change in diameter, $\delta d = ?$

Change in length, $\delta l = ?$

$$\text{Change in diameter}, \delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times (1250)^2}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right) = 0.25\text{mm}$$

$$\text{Change in length } \delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{1.5 \times 1250 \times 2500}{2 \times 20 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.12\text{mm}$$

CHAPTER:3

PRINCIPAL STRESS AND STRAIN

Q.1. Define principal plane and principal stress ***2006,(1-iii), 2010(1-c)***

Ans: At a point in a strained material, there are three mutually perpendicular plane, which carry only direct stress, no shear stress, is known as principal plane.

Principal Stress: the magnitude of the direct stress across the principal plane is known as principal stress.

Q2. Derive the principal stresses on a body subjected to two mutually perpendicular direct stresses accompanied with shear stresses

2012(w)1-(b), 2014(w)

Ans: Now let us consider an oblique section inclined with x-x axis an with we are required to find out stresses

Let σ_x = Tensile stress along x-x axis

σ_y = Tensile stress along y-y axis.

ζ = shear stress along x-x axis

θ = Angle which the oblique plane section AB.

First of all consider the equilibrium of the wedge ABC , ABC.

Horizontal force acting on the face AC,

$P_1 = \sigma_x . Ac\ (\rightarrow)$ ……………………. (1)

Vertical force acting on the face AC,

$P_2 = \zeta\ xy . AC\ (\downarrow)$ ………………… (2)

Similarly, vertical force acting on the face BC,

$P_3 = \sigma_y . BC\ (\downarrow)$ ……………………. (3)

Horizontal force on the face BC,

$P_4 = \zeta_{xy} . BC\ (\rightarrow)$ …………………….(4)

Now resolving the force perpendicular to the section AB

$P_n = P_s \sin\theta - P_2 \cos\theta + P_3 \cos\theta - P_4 \sin\theta$

$= \sigma_x . AC \sin\theta - \zeta_{xy} AC \cos\theta + \sigma_y . BC \cos\theta - \zeta_{xy} BC \sin\theta.$

Now resolving the force longentically to AB,

$P_1 = P_1 \cos\theta + P_2 \sin\theta - P_3 \sin\theta - P_4 \cos\theta$

$= \sigma_x . AC \cos\theta + \zeta_{xy} AC. \text{Sin}\,\theta - \sigma_y . BC \sin\theta - \zeta_{xy} BC \cos\theta.$

We know that normal stress across the section AB, $\sigma_n = p_n/AB$

$$= \frac{\sigma_x\ AC \sin\theta\ -\ \zeta_{xy}\ AC \cos\theta\ +\ \sigma_y\ BC \cos\theta\ -\ \zeta xy\, BC \sin\theta}{AB}$$

$$= \frac{\sigma_x\ AC \sin\theta}{AB} - \frac{\zeta_{xy}\ AC \cos\theta}{AB} + \frac{\sigma_y\ BC \cos\theta}{AB} - \frac{\zeta xy\, BC \sin\theta}{AB}$$

Q3. State the relation between maximum shear stress and principal shear stress at a point. 2006(w), 1(iv)

Ans:

$$= \frac{\frac{\sigma_x AC\sin\theta}{AC}}{\sin\theta} - \frac{\frac{\zeta_{xy}\ AC\cos\theta}{AC}}{\sin\theta} + \frac{\frac{\sigma_y\ BC\cos\theta}{BC}}{\cos\theta} - \frac{\frac{\zeta xy\ BC\sin\theta}{BC}}{\cos\theta}$$

$$= \sigma_x\ \sin^2\theta - \zeta_{xy}\sin\theta.\cos\theta + \sigma_y\ \cos^2\theta - \zeta xy\sin\theta.\cos\theta$$

$$= \frac{\sigma_x}{2} - (1-\cos 2\theta) + \frac{\sigma_y}{2}(1+\cos 2\theta) - 2\zeta_{xy}\sin\theta.\cos\theta$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta - \zeta_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \zeta_{xy}\sin 2\theta.................(5)$$

shear strss i.e. tan gential stress across the sec tion AB.

$$\zeta = \frac{pt}{AB}$$

$$= \frac{\sigma_x AC\cos\theta + \zeta_{xy}\ AC\sin\theta - \sigma_y\ BC\sin\theta - \zeta_{xy}\cos\theta}{AB}$$

$$= \frac{\sigma_x AC\cos\theta}{AB} + \frac{\zeta_{xy}\ AC\sin\theta}{AB} - \frac{\sigma_y\ BC\sin\theta}{AB} - \frac{\zeta_{xy}\cos\theta}{AB}$$

$$= \frac{\frac{\sigma_x AC\cos\theta}{AC}}{\sin\theta} + \frac{\frac{\zeta_{xy}\ AC\sin\theta}{AC}}{\sin\theta} - \frac{\frac{\sigma_y\ BC\sin\theta}{BC}}{\sin\theta} - \frac{\frac{\zeta_{xy}\ BC\cos\theta}{BC}}{\cos\theta}$$

$$= \sigma_x\sin\theta.\cos\theta + \zeta_{xy}\ \sin^2\theta - \sigma_y\ \sin\theta.\cos\theta - \zeta_{xy}\ \cos^2\theta$$

$$= (\sigma_x - \sigma_y)\sin\theta.\cos\theta + \frac{\zeta_{xy}}{2}(1-\cos 2\theta) - \frac{\zeta_{xy}}{2}(1+\cos 2\theta)$$

$$= \sigma_x - \sigma_y\sin 2\theta + \frac{\zeta_{xy}}{2} - \frac{\zeta_{xy}}{2}\cos 2\theta - \frac{\zeta_{xy}}{2} - \frac{\zeta_{xy}}{2}\cos 2\theta$$

$$= \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \zeta_{xy}\cos 2\theta.$$

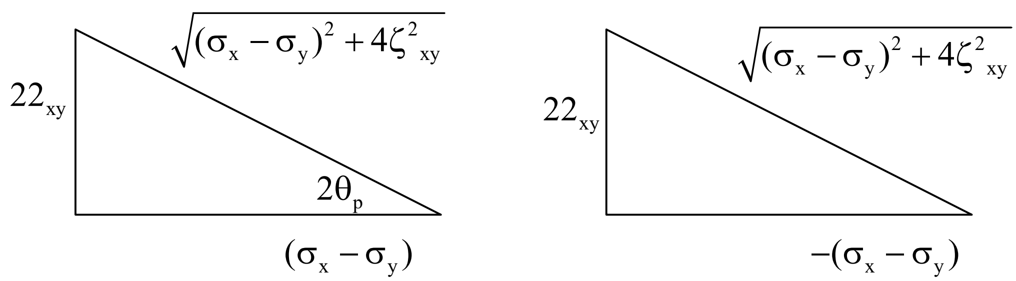
Now the principal stress acting on the principal planes may be found out by equating the on the shear stress to zero. Now let θ_p be the value of the angle for which the shear stress is zero.

$$\therefore \frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p - \zeta_{xy} \cos 2\theta_p = 0$$

$$\text{or} \frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p = \zeta_{xy} \cos 2\theta_p$$

$$\tan 2\theta_p = \frac{2\zeta_{xy}}{\sigma_x - \sigma_y}$$

From the above equation we find that the following two cases satisfy this condition as shown.



Thus we find that there are two principal planes at right angle to each other, their inclination with x-x axis being θ_p and $\theta_p{}^1$.

Now force case-1 we find that

$$\sin 2\theta_{p_1} = \frac{2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_1} = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\text{similarlyforcase} - 2$$

$$\sin 2\theta_{p_2} = \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_2} = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

Now the values of principal stress may be found out by substituting the above values of $2\theta_p$ and $2\theta^1_p$.

Maximum principal stress.

$$\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2}\cos 2\theta - \zeta_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\sigma_x + \sigma_y}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}} + \zeta_{xy} \times \frac{2\zeta_{xy}}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}.$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2{}_{xy}}$$

Minimum principal stress

$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)}}$$

$$+\zeta_{xy} \times \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{-(\sigma_x + \sigma_y)^2 - 2\zeta^2{}_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}}$$

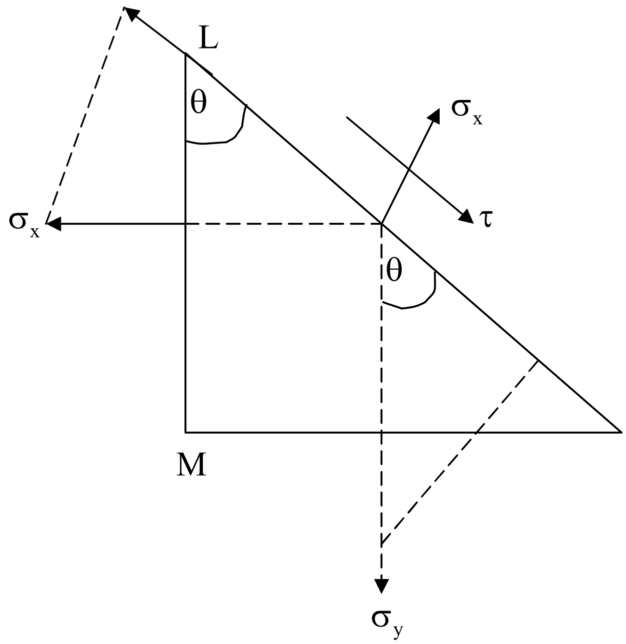
$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2{}_{xy}}$$

Problem

Derive an expression for stresses in two mutually perpendicular directions stresses. 2014(w), 3(b)

Consider that direct stresses σ_x and σ_y act across the faces LM and MN and that the block has unit depth perpendicular to LMN. Let the stresses τ and σ_n act on the same plane at an angle ‘θ’ to LM.



Resolving normal to LN.

$$\sigma_n \times LN = \sigma_x \times LM\cos\theta + \sigma_y MN\sin\theta$$

$$\sigma_n = \sigma_x \times \frac{LM}{MN}\cos\theta + \sigma_y.\frac{MN}{LN}\sin\theta$$

$$= \sigma_x.\cos^2\theta + \sigma_y.\sin^2\theta = \frac{\sigma_x}{2} \times 2\cos^2\theta + \frac{\sigma_y}{2} \times 2\sin^2\theta$$

$$= \frac{\sigma_x}{2}\left(1 - \sin^2\theta + \cos^2\theta\right) + \frac{\sigma_y}{2}\left(1 - \cos^2\theta + \sin^2\theta\right)$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sigma_x\left[\frac{\cos^2\theta - \sin^2\theta}{2}\right] - \sigma_y\left[\frac{\cos^2\theta - \sin^2\theta}{2}\right]$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2}\cos 2\theta$$

when $\upsilon = 0$ $\qquad \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} = \sigma_x$

when $\upsilon = \frac{\pi}{2}$ $\qquad \sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} = \sigma_y$

Resolving parallel to LN

$$\tau \times LN = \sigma_x \times LM \sin\theta - \sigma_y \times MN \cos\theta$$

$$\tau = \sigma_x \frac{LM}{LN} \sin\theta - \sigma_y \frac{MN}{LN} \cos\theta$$

$$= \sigma_x . \cos\theta . \sin\theta - \sigma_y . \sin\theta . \cos\theta = (\sigma_x - \sigma_y) \sin\theta$$

$$= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

The maximum value of τ occurs when

$$2\theta = \frac{\pi}{2} \qquad \text{or} \qquad \theta = \frac{\pi}{4}$$

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

$$\text{Resultant stress, } \sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

Problem:

The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive). Determine the magnitude of normal and shear stresses on a plane inclined at an angle of 25 with the tensile stress angle of 25^o with the tensile stress. Also determine the direction of resultant stress and magnitude of maximum intensity of shear stress 2012(w), 1(c).

Given:

$$\sigma_x = 100 MPa = 100 N/mm^2$$

$$\sigma_y = -50 MPa = -50 N/mm^2$$

$$\vartheta = 25^o$$

$$\text{Normal stress}, \sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$= \frac{100 + (-50)}{2} - \frac{100 - (-50)}{2} \times \cos 2 \times 25^o$$

$$= \frac{100 - 50}{2} - \frac{100 + 50}{2} \times \cos 50^o$$

$$= \frac{50}{2} - \frac{150}{2} \times \cos 50^o = 25 - 75 \cos 50^0 = -23.23 N/mm^2$$

$$\text{Shear stress}, \tau = \frac{\sigma_x - \sigma_y}{2} \times \sin 2\theta$$

$$= \frac{100-(-50)}{2} \times \sin 2 \times 25^o$$

$$= \frac{100+50}{2} \times \sin 50^o = 75 \sin 50^0 = 57.45 N/mm^2$$

Direction of Resultant stress

$$\tan\theta = \frac{\tau}{\sigma_n} = \frac{57.45}{-23.23} = -2.47$$

$$\Rightarrow \theta = \tan^{-1}(-2.47) = -68^o$$

Magnitude of maximum shear stress

$$T_{max} = \pm\frac{\sigma_x - \sigma_y}{2} = \pm\frac{100-(-50)}{2}$$

$$= \pm\frac{100+50}{2} N/mm^2 = \pm 75 N/mm^2$$

Problem:

A point in a strained material is subjected to a stress as shown below. Calculate principal stress ii) Maximum shear stress and also the plane along which and also the plane along which it acts. 2014(w),3(c)

Given :

$\sigma_x = 50$ MN/m^2

$\sigma_y = 100$ MN/m^2

$\tau = 25$ MN/m^2

$$\text{Major principal stress, } \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\frac{50+100}{2} + \sqrt{\left(\frac{50-100}{2}\right)^2 + (25)^2} = 110.35 MN/m^2$$

$$\text{Minor principal stress}, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{50+100}{2} - \sqrt{\left(\frac{50-100}{2}\right)^2 + (25)^2} = 39.65 MN/m^2$$

$$\text{Max. shear stress, } \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{50-100}{2}\right)^2 + (25)^2} = 35.35\,MN/m^2$$

Angle made by principal planes.

$$\tan 2\vartheta_p = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2\times 25}{50-100} = -1$$

or $\quad 2\vartheta_p = \tan^{-1}(-1) = 135^o$

or $\quad \vartheta_p = 67.5^o \quad$ or $\quad 157.5^o$

Q. Write short notes on Mohr's circle 2014(w)

Ans: We have already discussed analytical method for determination of various stresses across a section. Another method known as graphical method is used for determination of stresses. This is done by drawing a Mohr's circle of stresses.

The construction of Mohr's circle of stresses as well as determination of normal, shear and resultant stresses is very easier than the analytical method. More over there is a little chances of committing error in this method.

The angle is taken with reference to x-x axis. All the angles traced in anticlockwise direction to x-x axis are taken as negative where those in clockwise direction as positive. The value of angle 'υ' until and unless mentioned is taken as positive and drawn clock wise.

The measurement above x-x axis and to right of y-y axis is taken positive where as those below x-x axis and to left of y-y axis is taken negative.

Thus we find that there are two principal planes at right angle to each other, their inclination with x-x axis being θ_p and $\theta_p{}^1$.

Now force case-1 we find that

$$\sin 2\theta_{p_1} = \frac{2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_1} = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\text{similarlyforcase} - 2$$

$$\sin 2\theta_{p_2} = \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_2} = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

Now the values of principal stress may be found out by substituting the above values of $2\theta_p$ and $2\theta^1{}_p$.

Maximum principal stress.

$$\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2}\cos 2\theta - \zeta_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\sigma_x + \sigma_y}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}} + \zeta_{xy} \times \frac{2\zeta_{xy}}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}.$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2{}_{xy}}$$

$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)}}$$

$$+\zeta_{xy} \times \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{-(\sigma_x + \sigma_y)^2 - 2\zeta^2_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2_{xy}}$$

Problem:

A plane stress at a point is defined as σ_x = 20 MPa, σ_y = 40 MPa and z_{xy} =10 Mpa where the symbols have their usual meaning. Find thd principal stresses at the point and angles between principal planes. 2015(w),2(c)

Given :

$\sigma_x = 20$ MPa = 20 N/mm^2

$\sigma_y = 40$ MPa = 40 N/mm^2

$\tau = 10$ MPa = 10 N/mm^2

Manor principal stress, $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

$$\frac{20+40}{2} + \sqrt{\left(\frac{20-40}{2}\right)^2 + (10)^2} = \frac{60}{2} + \sqrt{\left(\frac{-20}{2}\right)^2 + (10)^2}$$

$$= 30 + 14.14 = 44.14 \text{N/mm}^2$$

Minor Principal stress $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

$$= \frac{20+40}{2} - \sqrt{\left(\frac{20-40}{2}\right)^2 + (10)^2} = \frac{60}{2} - \sqrt{\left(\frac{-20}{2}\right)^2 + (10)^2}$$

$$= 30 - 14.14 = 15.86 N/mm^2$$

Angle made by principal planes

$$\tan 2\vartheta_p = \frac{2\tau}{\sigma_x - \sigma_y} \Rightarrow 2\vartheta_p = \tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)$$

$$= \tan^{-1}\left(\frac{2\times 10}{20-40}\right) = \tan^{-1}(-1) = 135^o$$

$$\Rightarrow \vartheta_p = 67.5^o \quad \text{or} \quad 157.5^o$$

Problem : The principal stress at a point a bar are 200 N/mm^2 (tensile) and 100 N compressive. Determine the resultant stress in magnitude and direction on a plane inclined at 60^o to the axis of major principal stress. Find maximum intensity of shear stress in material at this point 2013(w)2c

Given :

$\sigma_x = 200$ N/mm^2

$\sigma_y = -100$ N/mm^2

$\theta = 60^o$

Normal stress,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{200 + (-100)}{2} - \frac{200 - (-100)}{2}\cos(2\times 60^o)$$

$$= \frac{200-100}{2} - \frac{200+100}{2}\cos 120^o = 50 - 150\cos 120^o = 125 N/mm^2$$

$$\text{shear stress}\ (\tau) = \frac{\sigma_x - \sigma_y}{2} \times \sin 2\theta$$

$$= \frac{200-(-100)}{2} \times \sin 2\times 60^o = 150 \sin 120^o = 129.9\, N/mm^2$$

$$\text{Resultant stress}, \sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

$$= \sqrt{(125)^2 + (129.9)^2} = 180.27 N/mm^2$$

Maximum intensity of shear stress

$$\tau_{max} = \pm\frac{\sigma_x - \sigma_y}{2} = \pm\frac{200-(100)}{2} = \pm 150 N/mm^2$$

CHAPTER:4

Shear force and bending moment

Q1. Define shear force and bending moment 2013,(6-a), 2014(w)

Ans: Shearing Force of the cross – section of a beam can be defined as the unbalanced vertical force to the left and right of the beam.

Bending Moment can be defined as the algebraic sum of moments of forces to the left or right of the cross-section of the beam.

Q.2. Draw S.F. and B.M. diagram of a cantilever loaded with U.d.L. spread over entire length 2007, 2(c)

Ans: Consider a section x at a distance x from B.

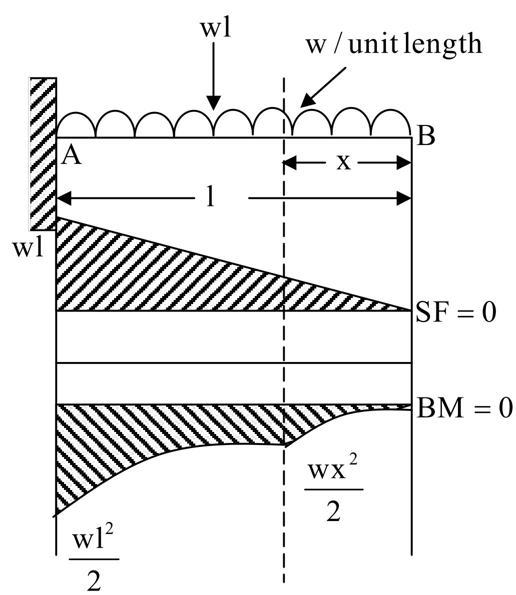
S.F. at B = F_B = 0

S.F. at x fx = wx

F_A = wl

BM: Mb = 0

Mx = wx × x/2 = $-wx^2/2$, MA = $-wl^2/2$



Q.3 Define contilever, or simple supported beam ***2006 1(vi)***

Ans: A beam fixed at one end and free of other and is known as contilever beam and a beam in which supports are situated at its two ends are known as simple supported beam.

Q.4. Define point of contraflexure.

Ans: The point, where the bending moment changes its sign or zero is known as point of contraflexure.

Q.5. Draw S.F. and B.M. diagram of a sample supported beam with U.K.L.

Ans: $R_n = R_B = wl/2 = 0.5\ wl$

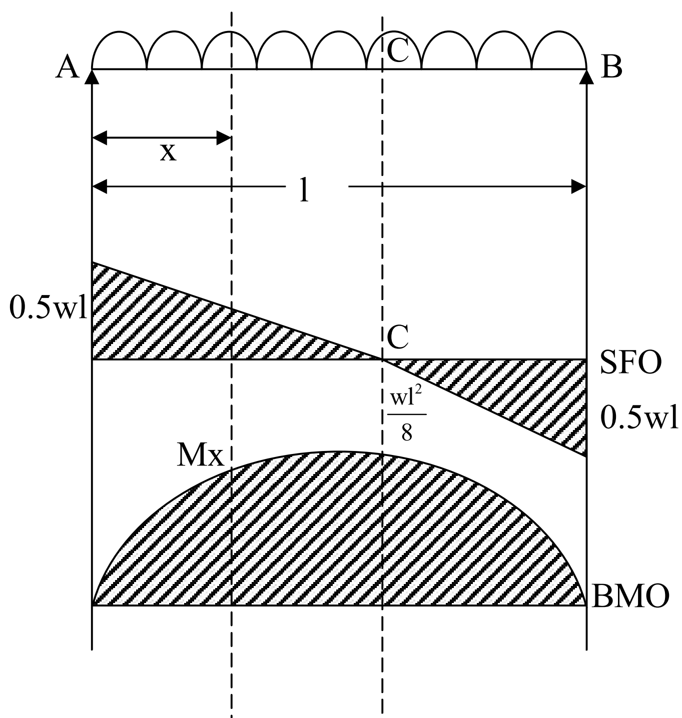
We know that SF at any section x at a distance x from A

$F_x = R_A - wx = 0.5\ wl - wx$

We know that BM at any section x from A

$Ln_x = R_{nx} - wx^2/2 = wlx/2 - wx^2/2$

$$Mc = \frac{wl}{2}\left(\frac{1}{2}\right) - \frac{w}{2}\left(\frac{1}{2}\right)^2 = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$$



Q. What are different types of beams. Explain in details.

2014(w),2015(w), 4(b)

Ans: The beams are classified as follows :

(i) Cantilever Beam

(ii) Simply supported Beam

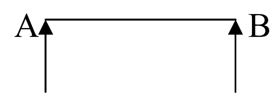
(iii) Over hanging Beam

(iv) Rigidly fixed or built – in – beam

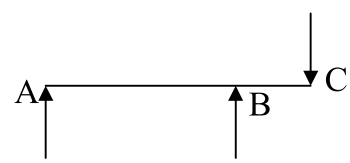
(v) Continuous beam.

Cantilever Beam: A beam fixed at one end and free at the other end is known as cantilever beam.

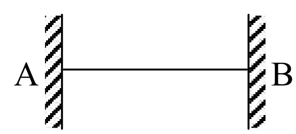
Simple supported beam: A beam supported at its two ends is known as simply supported beam



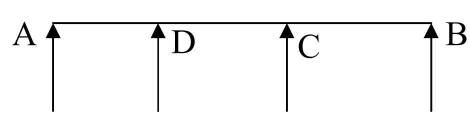
Over hanging beam: A beam which extends beyond its support is known as over hanging beam.



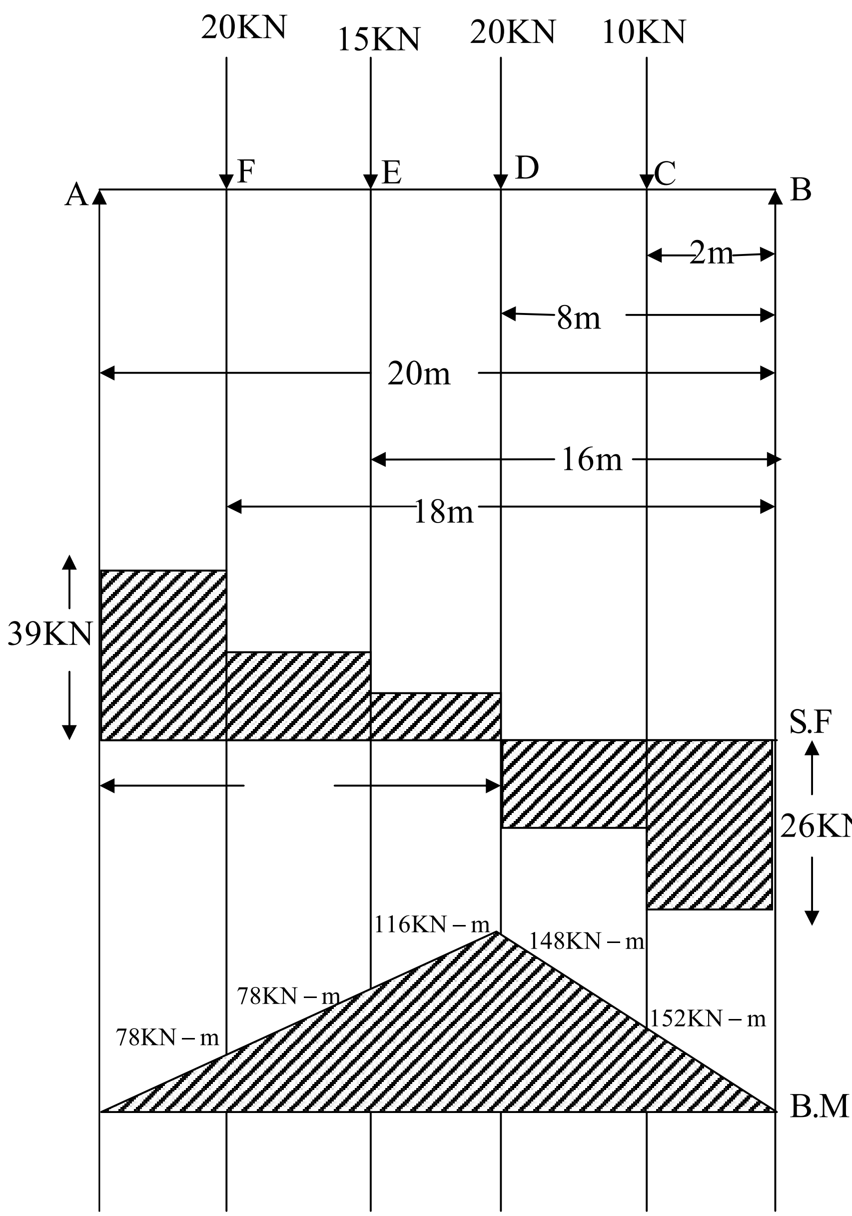
Rigidly fixed beam: A beam rigidly fixed at its both ends is known as rigidly fixed beam



Continuous Beam: A beam having more that two supports is known as continuous beam.



Problem: A horizontal beam of 20 m with simply supported at ends carries concentrated vertical loads of 10 KN, 20 KN, and 20 KN at 2m, 8m, 16 m and 18m respectively from right hand side of beam. Draw S.F. and B.M. diagram and find maximum S.F. and B.M.   2014(w) 4(c)



Let reaction at A = R_A reaction at B = R_B

Taking moment about A $R_B \times 20 = 10 \times 18 + 20 \times 12 + 15 \times 4 + 20 \times 2$

Or $20\ R_B = 180 + 240 + 60 + 40$ or $20\ R_B = 520$ KN.

Or $R_B = 520/20 = 26$ KN

$R_A + R_B = 20 + 15 + 20 + 10$

Or $R_A + R_B = 65$ KN

Or $R_A + 26 = 650$ or $R_A = 65 - 26 = 39$ KN.

Shear Force

S.F at A = $F_A = R_A = 39$ KN

S.F. at F = $F_f = 39 - 20 = 19$ KN

S.F. at E = $F_E = 19 - 15 = 4$ KN

S.F. at D, $F_D = 4 - 20 = -16$ KN

S.F. at C, $F_C = -16 - 10 = -26$ KN

S.F. at B, $F_B = -26$ KN.

Bending moment

B. M. at A, $M_A = 0$

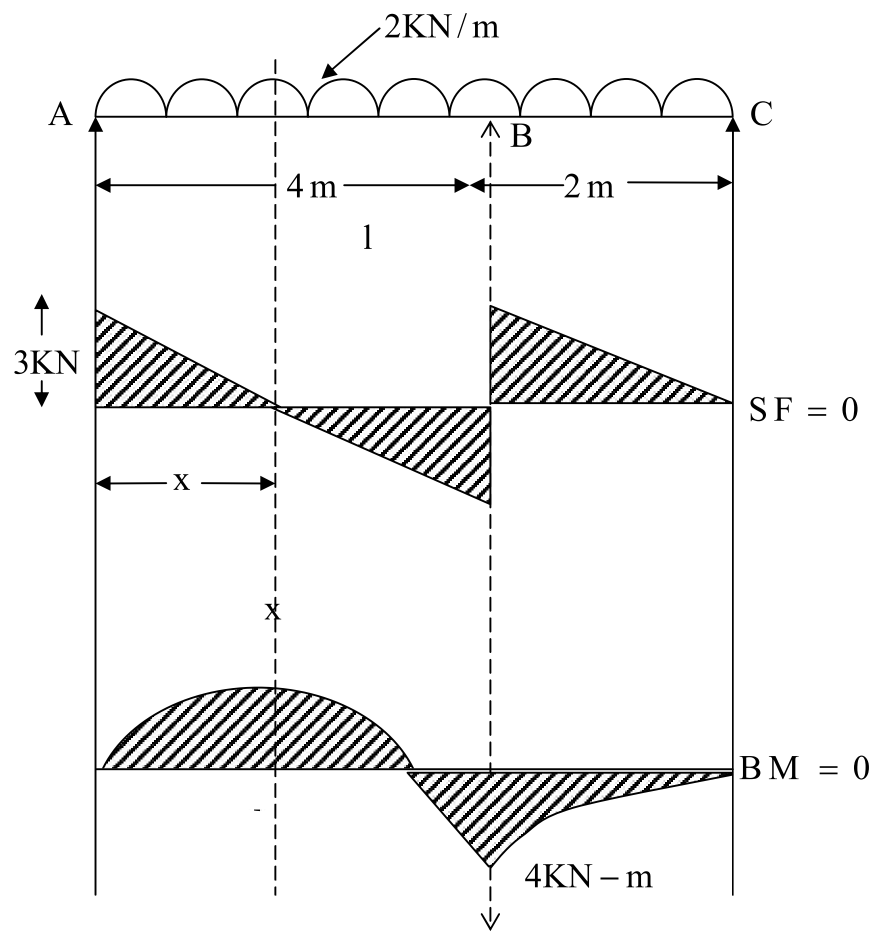
B.M. at F, $M_F = R_A \times 2 = 39 \times 2 = 78$ KN-m

B.M at E, $M_E = R_A \times 4 - 20 \times 2 = 39 \times 4 - 40 = 156 - 40 = 116$ KN-m

B.M at D, $M_D = R_A \times 12 - 20 \times 10 - 15 \times 8 = 39 \times 12 - 200 - 120 = 148$Kn-m

Problem :

Draw shear force and Bending moment diagram for an over hanging beam carrying U.D.L. of 2KN/m over entire span length of 6m as shown in figure. 2015(W), 5(C)



Let reaction at A = R_A

Relation at B = R_B

Taking moment about A $R_B \times 4 = 12 \times 3$ or $4\ R_B = 36$ KN

Or $R_B = 36/4 = 9$ KN

$R_A + R_B = 12$ KN.

Or $R_A + 9 = 12$ or $R_A = 12 - 9 = 3$ KN.

Shear force

S.F at A, $R_A - F_A = 3$ KN

S.F. just before B, $= 3 - 8 = -5$ KN

S.F. at B, $F_B = -5 + R_B = -5 + 9 = 4$ KN

S.F at C, $F_C = 4 - 4 = 0$

Bending moment will be maximum where S.F. is zero or changes sign consider a section 'X' at a distance x from A

S.F at X, $F_X = R_A - 2x = 3 - 2x$

$\therefore\ 3 - 2 = 0$ or $2x = 3$ or $x = 3/2 = 1.5$ m

Bending moment

B.M. at A, $M_A = 0$

B.M. at B, $M_B = R_A \times 4 - 2 \times 4 \times 2 = 3 \times 4 - 16 = -4$ KN-m

B.M. at C, $M_C = R_A \times 6 - 12 \times 3 + R_B \times 2$

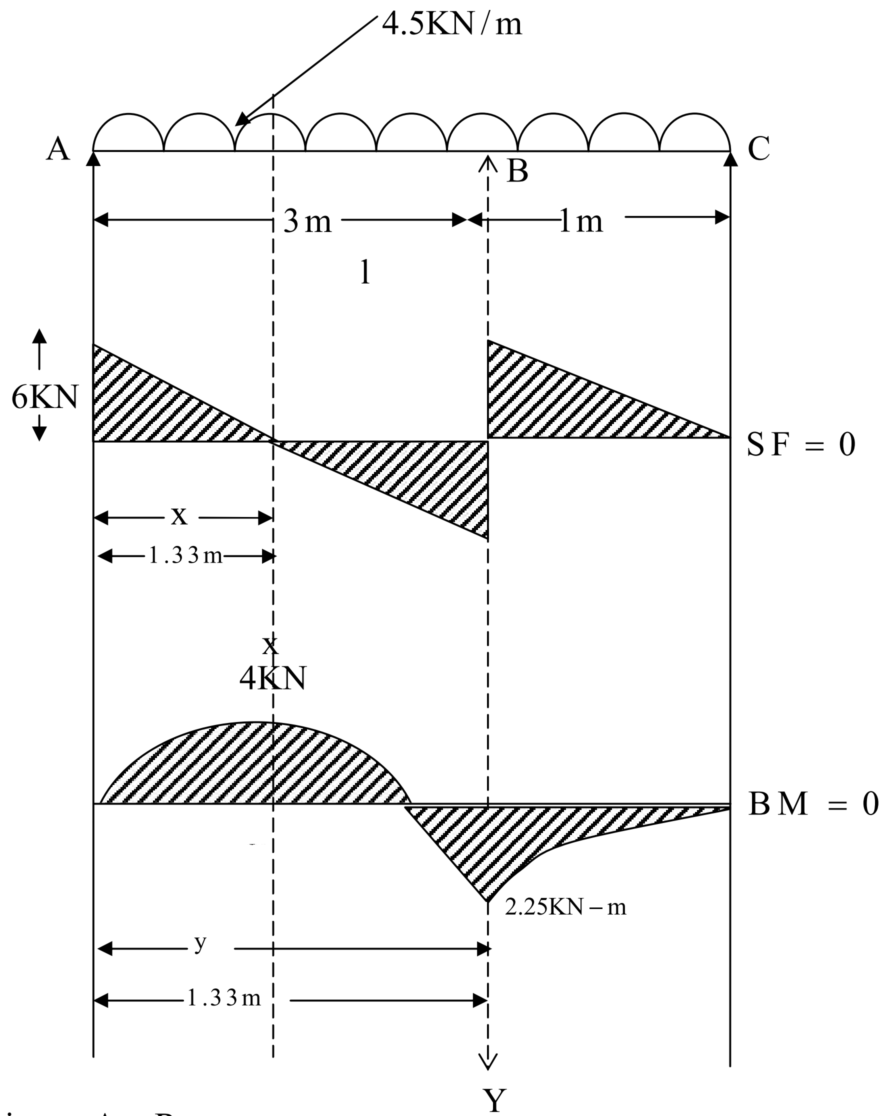
$= 3 \times 6 - 36 + 9 \times 2 = 0$

Maximum B.M, $M_X = R_A \times 1.5 - \dfrac{2 \times 1.5 \times 1.5}{2}$

$= 3 \times 1.5 - 2.25 = 4.5 - 2.25 = 2.25$ KN-m

Problem:

An overhanging beam ABC is loaded as shown in figure below. Draw S.F. and B.M. diagram and point of contraflexure. 2013(w),6(c)



Let reaction at A = R_A

Reaction at B = R_B

Taking moment about A

$R_B \times 3 = 12 \times 2$ or $3 R_B = 36$ KN or $R_B = 36/3 = 12$ KN.

$R_A + R_B = 18$ KN.

Or $R_A + 12 = 18$ or $R_A = 18 - 12 = 6$ KN.

Shear Force

S.F. at A, $F_A = R_A = 6$ KN.

S.F. just before B = 6 − 13.5 = − 7.5 KN.

S.F. at B, F_B = −7.5 + R_B = − 7.5 + 12 = 4.5 KN.

S.F at C, F_C = 4.5 − 4.5 = 0

Bending moment will be maximum at the point where S.F. ios either zero or changes sign

Consider a section 'X' at a distance x from A.

S.F. at X, $F_X = R_A - 4.5x = 6 - 4.5x$

$\therefore$ 6 − 4.5x = 0 or r.5x = 6 or x = 6/4.5 = 1.33 m

Bending moment

B.M. at A, $M_A = 0$

B.M. at B, $M_B = R_A \times 3\ (4.5 \times 3) \times 1.5 = 6 \times 3 - 13.5 \times 1.5$

$= 18 - 20.25 = -2.25$ KN-m

B.M. at C, $M_C = R_A \times 4 - 18 \times 2 + R_B \times 1$

$= 6 \times 4 - 36 + 12 \times 1 = 0$

Max. B.M., $M_X = R_A \times 1.33 - 4.5 \times 1.33 \times 1.33/2, = 5 \times 1.33 - 4.5 \times 1.33 \times 1.33/2 = 4$KN-m

Consider a section 'Y' at a distance y from A

$$\text{B.M. at Y, } M_Y = R_A \times y - 4.5 \times y \times \frac{y}{2} = 6y - \frac{4.5y^2}{2}$$

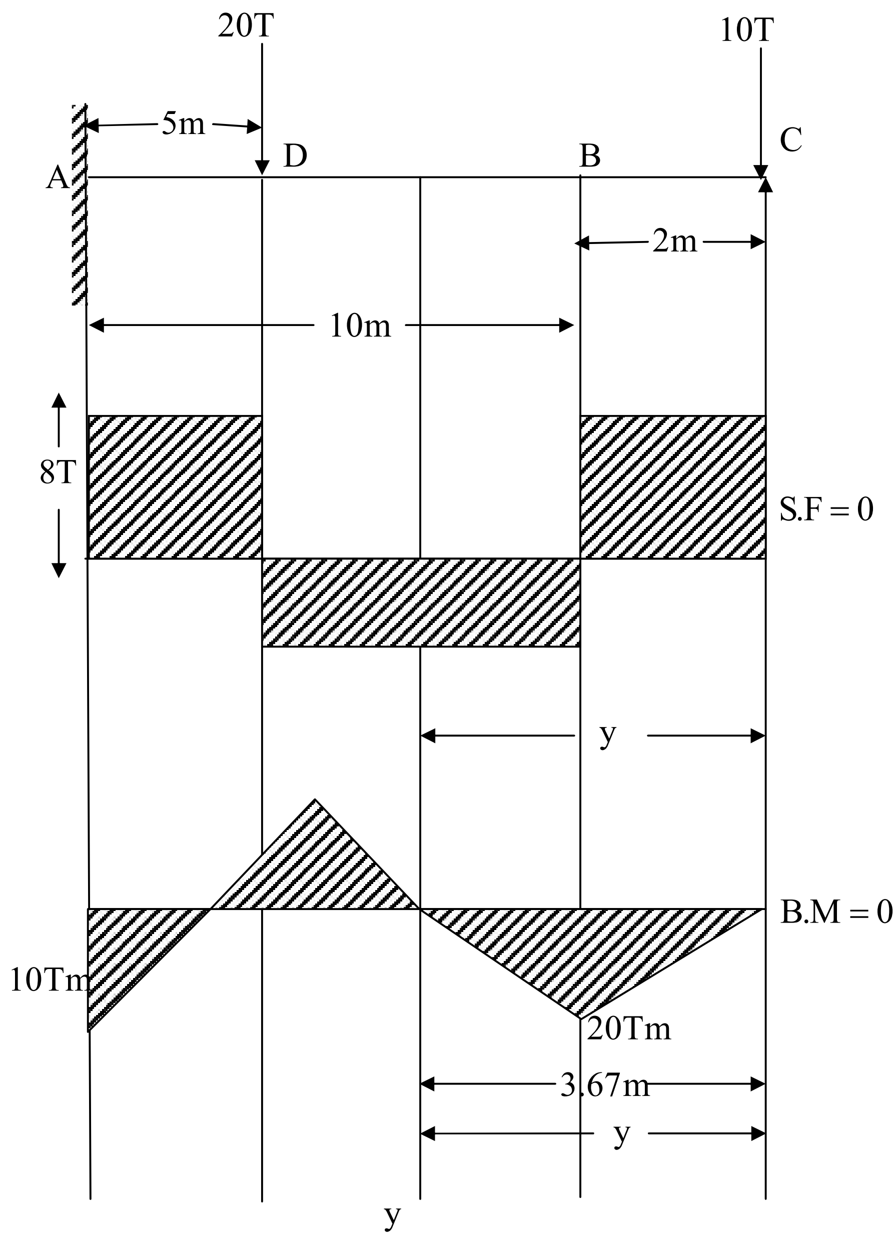
$$\therefore 6y - \frac{4.5y^2}{2} = 0$$

$$\text{or} \quad \frac{4.5y^2}{2} = 6y \quad \text{or} \frac{4.5y}{2} = 6$$

$$\text{or} \quad 4.5\,y = 12 \quad \text{or} \quad y = \frac{12}{4.5} 2.67\text{m}$$

Problem:

A cantilever beam 12 m long overhangs one side with simple support 12 m loaded as shown. Draw S.F. and B.M. diagram and find point of contraflexure. 2014(w), 5(c)



Taking moment about A

$R_B \times 10 = 10 \times 12 + 20 \times 5$ or $10\ R_B = 120 + 100$ or $10\ R_B = 220$ T

Or $R_B = 220/10 = 22$T

$R_A + R_B = 20 + 10$ or $R_A + R_B = 30$ T

Or $R_A + 22 = 30$

Or $R_A = 30 - 22 = 8$T

Shear Force

S.F at C, $F_C = 10$ T

S.F at B, $F_B = 10 - 22 = -12$T

S.F at D, $F_D = -12 + 20 = 8$T

S.F at A, $F_A = 8$T

Bending moment

B.M. at C, $M_C = 0$

B.M. at B, $M_B = -10 \times 2 = -20$T-m

B.M. at D, $M_D = -10 \times 7 + R_B \times 5 = -70 + 22 \times 5 = 40$ T-m

B.M. at A, $M_A = -10 \times 12 + 22 \times 10 - 20 \times 5 = -120 + 210 - 100 = -10$ KN-m

Consider a section Y at a distance y from C

B.M. at Y, $M_Y = -10 \times y + R_B\ (y - 2) = -10y + 22\ (y-2)$

$= -10y + 22y - 44 = 1y - 44$

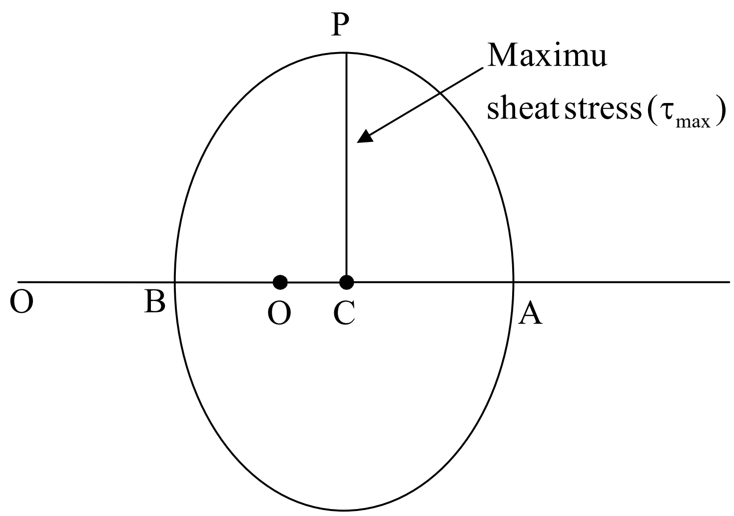
$\therefore\ 12y - 44 = 0$ or $12y = 44$

Or $y = 44/12 = 3.67$ m

Problem :

Draw Mohr's circle of point subjected direct stresses 200 N/mm^2 (tensile) and 100 N/mm^2 (compressive) along x and y are respectively. What is maximum shear stress 2014(w), 1(c).

Ans: Take scale as 20 N/mm^2 = 1 cm



First order a line OA = 10 cm and OB = 5 cm. Bisect AB at point C. Taking CA or CB as radius and C as centre draw a circle passing through A and B. At C draw a perpendicular meeting the circle at point P. CP is the maximum shear stress. Measure the length of CP in centimeter and convert it into required value of shear stress by multiplying 20 N/mm^2 with it.

Problem:

Find out S.F. and B.M. diagram at sailent points of the following loaded Beam and draw S.F. and B.M. diagrams 2014(w) ,4(c)

Ans: Taking moment about A

$R_B \times 8 = 15 \times 6.5 + 25 \times 2 + 10 \times 10$

$\Rightarrow 8\ R_B = 97.5 + 50 + 100$

$\Rightarrow 8\ R_B = 247.5$ KN

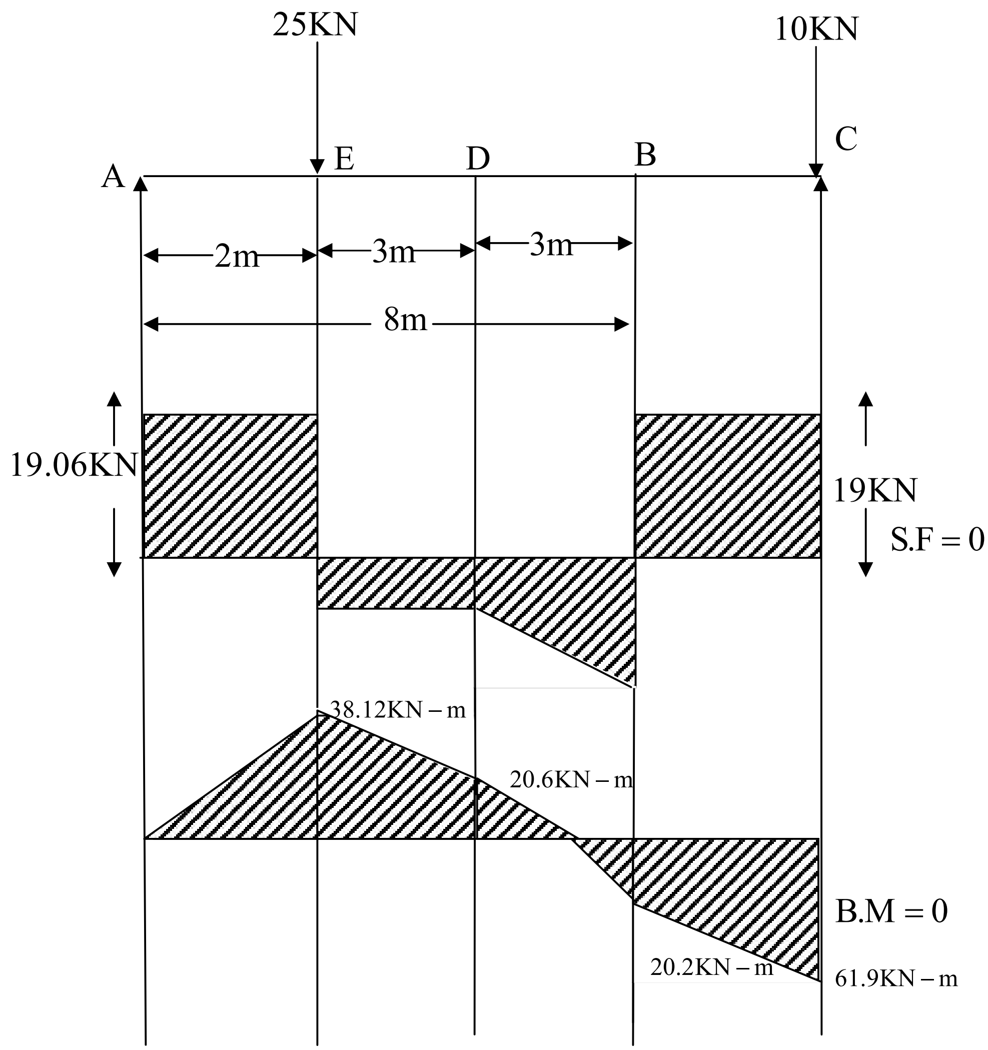
$\Rightarrow R_B = 247.6/8 = 30.94$ N

$R_A + R_B = 25 + 15 + 10$

$\Rightarrow R_A + R_B = 50$ KN

$\Rightarrow R_A + 30.94 = 50$

$\Rightarrow R_A = 50 - 30.94 = 19.06$ KN



Shear Force:

S.F. at A, $F_A = 19.06$ KN

S.F. at B, $F_B = 19.06 - 25 = -5.94$ KN

S.F at D, $F_D = -5.94$ KN

S.F just before B = $-5.94 - 15 = -20.94$ KN

S.F. at B, $F_B = -20.94 + R_B = -20.94 + 30.94 = 10$ KN

S.F. at C, $F_C = 10$ KN

Bending moment

B.N. at A, $M_A = 0$

$M_E = R_A \times 2 = 19.06 \times 2 = 38.12$ KN-m

$M_D = R_A \times 5 - 25 \times 3 = 19.06 \times 5 - 75 = 20.3$ KN-m

$M_B = R_A \times 8 - 25 \times 6 - 15 \times 1.5 = 19.06 \times 8 - 150 - 22.5 = -20.02$ KN-m

$M_C = R_A \times 10 - 25 \times 8 - 15 \times 3.5$

$19.06 \times 10 - 25 \times 8 - 15 \times 3.5 = 190.6 - 200 - 52.5 = -61.9$ KN-m

Problem

An overhanging beam ABC is loaded as shown raw S.F. and B.M. diagram and Find point of contraflexure (2011,2012,2013) (6-c)

Ans:

Taking moment about A and equating it same $R_B \times 3 - (4.5 \times 4) \times 2 = 36$

$R_B = 36/3 = 12$ KN

$R_A = (4.5 \times 4) - 12 = 6$ KN

S.F. diagram

$F_A = R_A = 6$ KN

$F_B = 6 - (4.5 \times 3) + 12 = 4.5$ KN

$F_C = 4.5 - (4.5 \times 1) = 0$

B.N. diagram

$M_A = 0$

$M_B = -(4.5 \times 1 \times ½) = -2.24$ KN

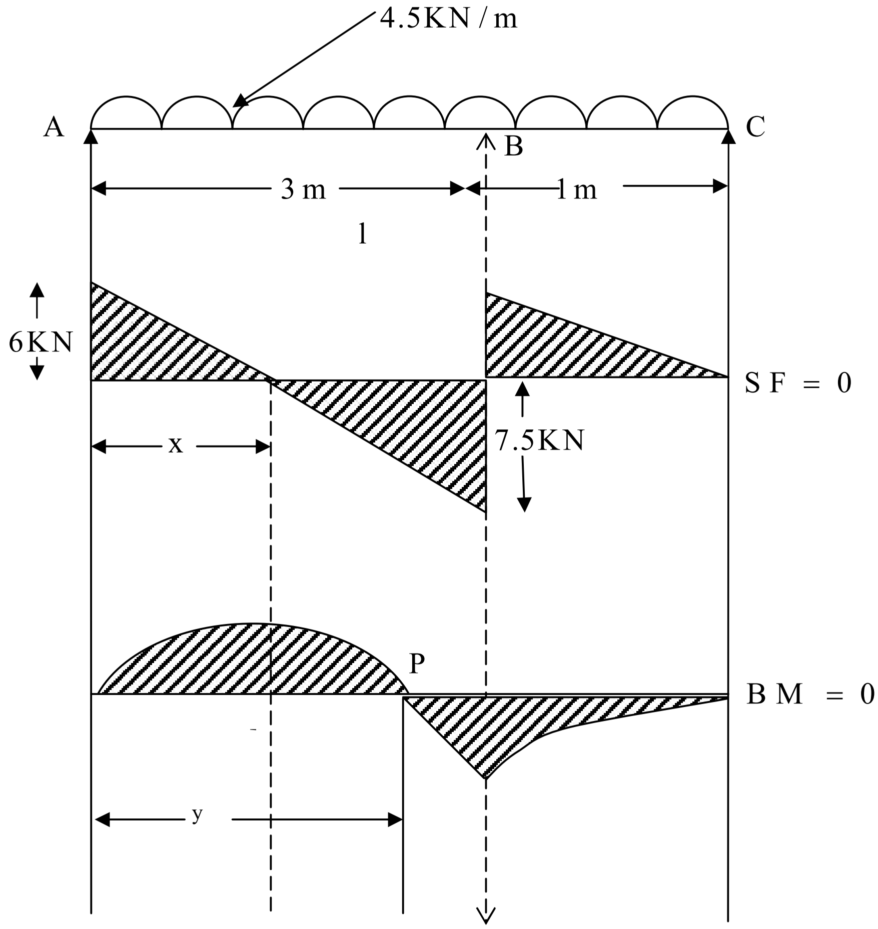
$M_C = 0$

Maximum B.M. will occur at M where S.F. changes sign. From the geometry of figure between A and B

$$\frac{x}{6} = \frac{3-x}{7.5} \text{or} 7.5x = 18 - 6x.$$

$$\text{or} 13.5x = 18 \text{or } x = \frac{18}{13.5} = 1.33\text{m}$$

$$M_M = (6 \times 1.33) - 4.5 \times 1.33 \times \frac{1.33}{2} = 4\text{KN} - \text{m}$$



Point of contraflexure.

Let 'P' be the point of contraflexure at a distance y from support A

$M_P = 6 \times y - 4.5\ y \times y/2 = 0$

Or $2.25\ y^2 - 6y = 0$

Or $2.25\ y = 6$ or $y = 6/2.25 = 2.67$ m

B.M. at C, $M_C = R_A \times 18 - 15 \times 14 - 20 \times 6 - 20 \times 16$

$= 39 \times 18 - 210 - 120 - 320 = 702 - 65 = 52$ KN

B.M at B, $M_B = R_A \times 20 - 20 \times 18 - 15 \times 16 - 20 \times 8 - 10$

$= 39 \times 20 - 20 \times 18 - 15 \times 16 - 20 \times 8 - 10 \times 2$

$= 780 - 360 - 240 - 160 - 20 = 0$

Maximum B.M. = 148 Kn-m

Maximum S.F. = 39 Kn

Q. What is sagging Bending moment and hogging Bending moment

2013 (w)

Ans:

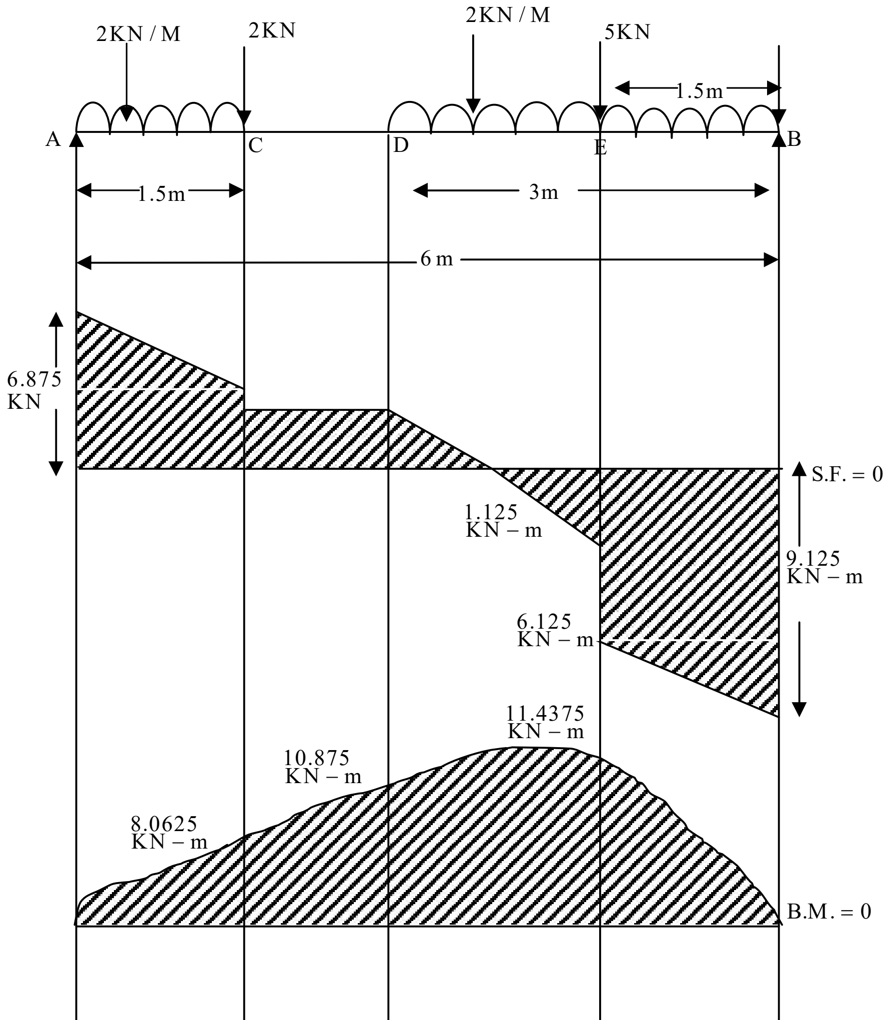
Sagging B.M. :- Positive Bending moment is known as sagging B.M. A bending moment is called sagging B.M. if it tends to bend the beam at a point to a curvature having concavity at top.

Hogging B.M.: Negative Bending moment is known as hogging B.M. A bending moment is called hogging B.M. if it tends to bend the beam at a point to a curvature having convexity.

Problem:

A simply supported beam AB 6m long is loaded as shown in figure below. Draw S.F. and B.M. diagram for the beam 2012(w), 4(c)

Ans:



Let relation at A = R_A

Relation at B = R_B

Taking moment about A,

$R_B \times 6 = 5 \times 4.5 + 6 \times 4.5 + 2 \times 1.5 + 3 \times 0.75$

Or $6R_B = 22.5 + 27 + 3 + 2.25$

Or $6R_B = 54.75$ or $R_B = 54.75/6 = 9.125$ Kn

$R_A + R_B = 3 + 2 + 6 + 6 = 16$ KN

$R_A = 16 - R_B = 16 - 9.125 = 6.875$ KN

Shear Force

S.F. at A $F_A = R_A = 6.875$ KN

S.F. just before C = 6.875 − 3 = 3.875 KN

S.F. at C, $F_C = 3.875 - 2 = 1.875$ KN

S.F. at D, $F_D = 1.875$ KN

S.F. just before E = 1.875 − 3 = −1.125 KN

S.F. at E, $F_E = -1.125 - 5 = -6.125$ KN

S.F. at B, $F_B = -6.125 - 3 = -9.125$ KN.

Bending Moment

B.M. at A, $M_A = 0$

B.M. at C, $M_C = R_A \times 1.5 - 3 \times 0.75 = 6.875 \times 1.5 - 2.25 = 8.0625$ KN-m

B.M. at D, $M_D = R_A \times 3 - 2 \times 1.5 - 3 \times 2.25$

$= 6.875 \times 3 - 3 - 6.75 = 20.625 - 9.75 = 10.875$ KN-m

B.M. at E, $M_E = R_A \times 4.5 - 2 \times 3 - 3 \times 3.75 - 3 \times 0.75$

$= 6.875 \times 4.5 - 6 - 11.25 - 2.25$

$= 30.9375 - 19.5 = 11.4375$ Kn-m

B.M. at B, $M_B = R_A \times 6 - 3 \times 5.25 - 2 \times 4.5 - 5 \times 1.5 - 6 \times 1.5$

$= 6.875 \times 6 - 15.75 - 9 - 7.5 - 9$

$= 41.250 - 41.250 = 0$

CHAPTER:5

Q. Write short notes on section modulus: 2014, 6(ii)

Ans: The section modulus of a beam is the ratio of moment of inertia of section to the distance of extreme compressive fibre from neutral axis.

It plays an important role in design of beams. It is the direct measure of strength of beam. A beam having larger section modulus wll be stronger and can support greater load. It is denoted by Z.

Section modulus, Z = I/y

Where I = Moment of Inertia

Y = distance from the centroid to top or bottom edge of section

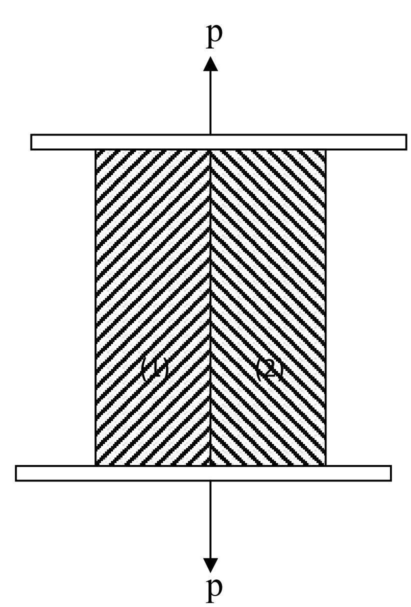
To calculate (z) the distance (y) to the extreme fibre from the centroid must be found as that is where maximum stress could cause failure.

For symmetrical sections the distance of extreme compressive fibre from neutral axis i.e. y_{max} and y_{min} are equal. But in case of unsymmetrical section the section modulus used will be differ depending on whether the compression occurs in the web or flat of section

Q. What is composite bar. State the conditions for analysis of composite bar. 2015, (3-b)

Ans: A composite bar is made up of two or more different materials joined together and the constituent materials have different properties.

Composite bars put in parallel and rigidly fixed with each other act as a single piece for the extension or contraction of constituent material when subjected to an axial tensile or compressive load.



Condition for analysis of composite bar

(i) The extension or contraction of constituent material is same and strain in all constituent material is same.

(ii) The total external load applied on composite bar is equal to the sum of loads carried by bars of different materials.

(iii) If the lengths of two bars are different the elongation should be calculated separately and equated total load on composite bar $P = P_1 + P_2$.

Strain in bar 1 = strain in bar 2 i.e. $l_1 = l_2$

Or $p_1/E_1 = p_2/E_2$

Where p_1 = load carried by bar 1

P_2 = load carried by bar 2

E_1 = Young's modulus of bar 1

E_2 = Young's modulus of bar 2

Q. Write short notes on resilience, 2014(w) 6(i)

Ans: Whenever a lead is applied on an elastic material, some deformation takes place. The body offers resistance against this deformation. The resistance offered by the body per unit area is termed as stress.

It should be noted that stress will only be developed in the body when the body has the ability to offer resistance to deformation. So when an elastic body is deformed due to applied load some work is done due to this deformation. On removing the load the body comes back to its original position. This energy which is stored in the body when strained within elastic limit is known as strain energy.

So long as the body remains loaded within elastic limit it stores energy which is known as strain energy which is known as strain erergy or resilience. This strain energy is capable of doing work.

$\therefore$ strain energy = work done

Q. What are the assumptions of pure bending ? ***2014(w), 5(b)***

Ans: Assumptions of pure Bending.

i) The material of beam is perfectly homogeneous i.e. of equal elastic properties in all directions.

ii) the beam material is stressed within the elastic limit and thus obeys Hooke's law.

iii) the transverse sections which were plane before bending remains plane after bending.

iv) Each layer of beam is free to expand or contract independently of layer above or below it.

v) the value of E(Young's modulus of elasticity is same in tension and compression.

vi) The beam is in equilibrium i.e. there is no resultant pull or push in beam section.

Q. Define moment of resistance and flexural rigidity. ***2014(w), 5(a)***

Ans: Moment of resistance. The moment of couple which resists the external bending moment is known as moment of resistance.

Q. Define flexural idigity ***2014(w)***

Ans: Flexural rigidity. The quantity EI in expressions for beam deflection is known as flexural rigidity. Where E = Young's modulus

I = Moment of inertia of beam section

Q. Define point load and U.D.L. 2015(w), 4(a)

Ans: Point Load: A point load is one which is considered to act at a point/

Uniformly distributed load (U.D.L.): A uniformly distributed load is one which is distributed at the uniform rate over the length of beam and is abbreviated as U.D.L.

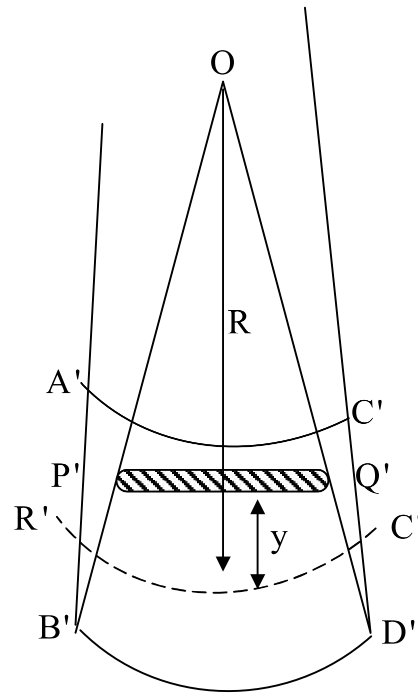
THEORY OF SIMPLE BENDING

Q1. What do you mean by pure bending ?2012 (5-a), 2014(w), 2015(w),(5-a)

Ans: A beam can be subjected to either axial force, shearfore, bending force or tension. But when a beam is only subjected to bending moment then, it is known as pure bending.

Q.2. Derive the relation $\frac{\sigma}{y}=\frac{E}{R}=\frac{M}{I}$

2012,5(c),2013,7(b), 2003, (3) 2010, 2(f)



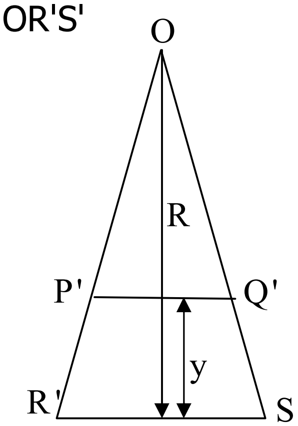
Ans: Consider a small section of a beam with RA' be the neutral axis. Now consider a small unit layer PQ. As the beam is subjected to bending moment. So, the layer PQ will bend to P'Q'.

We know that strain of the layer PQ

$$\in=\frac{\delta}{l}=\frac{\text{change in length}}{\text{origihnal length}}=\frac{PQ-P'Q'}{PQ}$$

Now consider the Δ OP'Q' and OR'S' & OP'Q' ~ OR'S'

so,



$$\frac{P'Q'}{R'S'} = \frac{R-y}{R}$$

or $$1-\frac{P'Q'}{R'S'} = 1-\frac{R-y}{R} \text{(Subtracting 1)}$$

$$\Rightarrow \frac{R'S'-P'Q'}{R'S'Q} = \frac{R-R+y}{R}$$

$$\Rightarrow \frac{PQ-P'Q'}{PQ} = \frac{y}{R} (\because PQ = R'S')$$

Q. Define section modulus and polar modulus. ***2015, 6(a)***

Ans: Section modulus : It is the ratio between moment of inertia and distance from neutral fibre to extreme fibre section modulus, z = I/y.

Polar modulus: It is the ratio between polar moment of Inertia to radius of polar modulus $Z_P = I_P/R$

Q3. Define section modulus. 2013(w),1(vi), 2006,1(v), 2007,1(viii),2005,1(e)

Ans: It is the ratio between the moment of inertia about neutral axis to the distance of most distant point from neutral axis.

Section Modulus Z = I/y.

Q. What is flexural rigidity ?

Ans: The expression EI for beam deflection is known as flexural rigidity.

Problem:

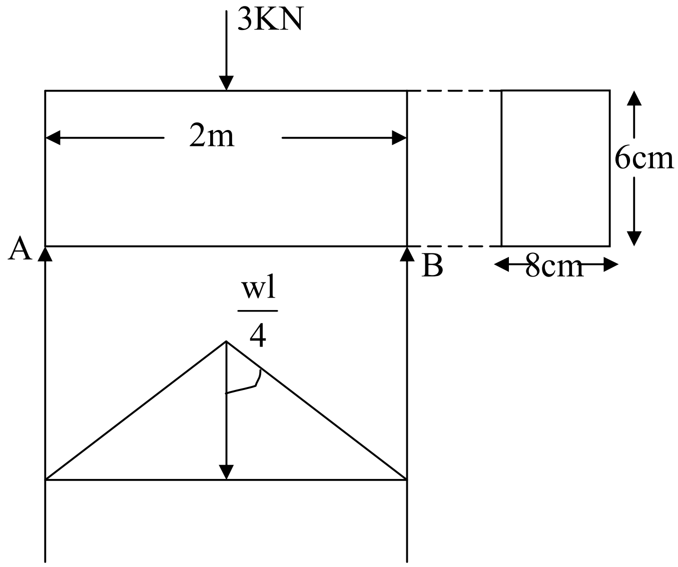
A rectangular beam 8 cm × 6 cm is 2m long and is simply supported at the ends. It carries a load of 3 KN at its midspan. Determine the maximum bending stress induced in the beam. 2014(w), 5(c)

Ans: Width of beam b = 8 cm = 80 mm

Depth, d = 6 cm = 60 mm

Load at midspan W = 3 KN

Maximum bending moment



$$M_{max} = \frac{wl}{4} = \frac{3 \times 2}{4} = 1.5\,KN - m = 1.5 \times 10^6\,N - mm$$

$$M.I\ of\ beam\ section, I = \frac{bd^3}{12} = \frac{80 \times (60)^3}{12}$$

$$= 1440000 m^4$$

$$y = \frac{d}{2} = \frac{60}{2} = 30mm.$$

Using bending equation.

$$\frac{M}{I} = \frac{(\sigma_b)max}{y}$$

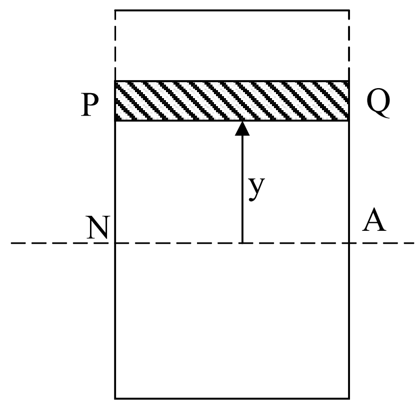
$$\Rightarrow (\sigma_b)max = \frac{M}{I} \times y = \frac{1.5 \times 10^6}{1440000} \times 30 = 31.25\,N/m$$

$$\in = \frac{y}{R}\left(\because \frac{PQ - P'Q'}{PQ} = \in\right)$$

$$\frac{\sigma_b}{E} = \frac{y}{R} \qquad \text{(By 400 Hook's law)}$$

$$\Rightarrow \sigma = \frac{E}{R}.y \quad \text{or} \quad \frac{\sigma_b}{y} = \frac{E}{R} ------ (i)$$

Since also we know that consider a small layer of a beam be PQ with NA as the neutral axis. Let δa= Area of the small layer PQ.



Intensity of stress in the layer PQ

$\Rightarrow \quad \sigma = E/R.\ y$

Total stress in the layer PQ

$$\Rightarrow \sigma = \frac{E}{R}.y \times \delta a.$$

Now moment of inertia of this total stress about NA

= Force × distance

$$= \sigma = \frac{E}{R}.y \times \delta a \times y$$

$$\Rightarrow \frac{E}{R}.y^2.\delta a$$

Moment of all such layers about NA will be equal to the moment of resistance (M).

$$\Rightarrow M = \sum \frac{E}{R} y^2 \delta a$$

$$\Rightarrow M = \frac{E}{R} y^2 \delta a$$

$$\Rightarrow M = \frac{E}{R}.I \quad \left(\because \sum y^2 \delta a = I\right)$$

$$\Rightarrow M = \frac{E}{R} - - - - - - - - - (ii)$$

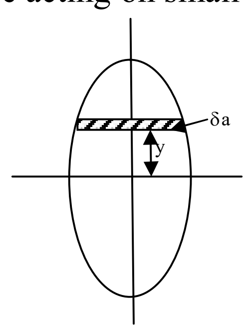
Now equation (i) and equation (ii) are equal

$$\therefore \frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$$

Problem:

Prove that neutral axis in a loaded beam is the centroidal axis. 2015(w),5(b)

Ans: Consider the cross section of a beam. There will be no resultant force on the section for condition of equilibrium. The force acting on small area δa at a distance y from neutral axis is given by



$\delta F = \sigma.\delta a = E/R \, . \, y \, . \, \delta a$

Or Total force normal to section

$$F = \frac{E}{R}, \sum y, \delta a$$

∴ For zero resultant force Σy, δa = 0. Now Σy, δa is the moment of sectional area about the neutral axis and since this moment is zero the axis must pass through centre of area. Hence the neutral axis passes through centre of area.

CHAPTER:7

Problem: Determine the diameter of solid shaft which will transmit 90 KW at 160 rpm if the shear stress in the shaft is limited to 60 N/mm^2. Find also the length of shaft if twist must not exceed 1 degree over the entire length. Take $c = 8 \times 10^4$ N/mm^2 2013(w), 7(c)

Given $P = 90$ KW $= 90 \times 10^3$ W

$N = 160$ rpm

$\tau = 60$ N/mm^2

$\upsilon = 1^o = 1 \times \pi/180$ radian $= \pi/180$ radian

$C = 8 \times 10^4$ N/mm^2

$$P = \frac{2\pi NY}{60} \Rightarrow \qquad T = \frac{P \times 60}{2\pi N} N - m$$

$$= \frac{90 \times 10^3 \times 60}{2 \times \pi \times 160} N - m = 53715 N - m$$

$$= 53715 \times 10^3 N - mm$$

$$T = \frac{\pi}{16} \times \tau \times d^3 \Rightarrow \quad d = \sqrt[3]{\frac{16 \times 53715 \times 10^3}{\pi \times 60}} = 77mm$$

$$\text{Polar moment of inertia}, J = \frac{\pi}{32} \times d^4$$

$$= \frac{\pi}{32} \times (77)^4 = 3451142 \, mm^4$$

$$\frac{T}{J} = \frac{C\vartheta}{L}$$

$$\Rightarrow \qquad \frac{53715}{3451142} = \frac{8 \times 10^4 \times \pi}{180 \times L}$$

$$\Rightarrow \qquad L = \frac{8 \times 10^4 \times \pi \times 3451142}{53715 \times 180} = 897 \, mm$$

Problem : What diameter of shaft will be required to transmit 80 KW at 80 rpm if maximum torque is 30 percent more than the mean and limit of torsional stress is to be 56 MPa. 2014(w), 7(c)

Given $P = 80\ KW = 80 \times 10^3 W$

$N = 80$ rpm

$T_{max} = 1.3 \times T_{mean}$

$\tau = 56$ Mpa $= 56\ N/mm^2$

Let d = diameter of shaft.

Mean torque, $T_{mean} = \dfrac{P \times 60}{2\pi N} N-m$

$$= \frac{80 \times 10^3 \times 60}{2 \times \pi \times 80} N-m = 9554\, N-m$$

$$= 9554 \times 10^3\, N-mm$$

Maximum torque, $T_{max} = 1.3 \times T_{mean}$

$= 1.3 \times 9554 \times 10^3\, N-mm$

$= 12420.2 \times 10^3\, N-mm$

$$T_{max} = \frac{\pi}{16} \times \tau \times d^3$$

$$\Rightarrow 12420.2 \times 10^3 = \frac{\pi}{16} \times 56 \times d^3$$

$$\Rightarrow d = \sqrt[3]{\frac{12420.2 \times 10^3 \times 16}{\pi \times 56}} = 104mm$$

Polar moment of Inertia, $J = \frac{\pi}{32} \times d^4$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\Rightarrow \frac{5307.86 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{80 \times 10^3 \times \pi}{180 \times 3000}$$

$$\Rightarrow d^4 = \frac{5307.86 \times 10^3 \times 32}{\pi \times d^4} = \frac{80 \times 10^3 \times \pi}{180 \times 3000}$$

$$\Rightarrow d^4 = \frac{5307.86 \times 10^3 \times 32 \times 180 \times 3000}{\pi \times 80 \times 10^3 \times \pi}$$

$$= 365047770.7$$

$$\Rightarrow d = \sqrt[4]{365047770.7} = 105.8\text{mm}$$

taking larger value

diameter of shaft d = 105.8 mm say 106 mm

Problem:

A shaft is transmitting 100 KN at 180 rpm. The allowable shear stress in shaft material is 60 MPa. Determine suitable diameter of shaft. The shaft is not to twist more than 1^o in a length of 3 metres. Take modulus of rigidity = 80 GPa. 2012(w), 7(b)

Given $P = 100\ KW = 100 \times 10^3\ W$

$N = 180$ rpm

$\tau = 60$ Mpa $= 60\ N/mm^2$

$$\vartheta = 1^0 = 1 \times \frac{\pi}{180} \text{radian} = \frac{\pi}{80} \text{radian}$$

$L = 3m = 3000$ mm

$C = 80\ GPa = 80 \times 10^3\ N/mm^2$

Let d = diameter of shaft

Torque transmitted by shaft.

$$T = \frac{P \times 60}{2\pi N} N - m = \frac{100 \times 10^3 \times 60}{2 \times \pi \times 180} N - m$$

$$= 5307.86 N - m = 5307.86 \times 10^3 N - mm$$

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$\Rightarrow \quad 5307.86 \times 10^3 = \frac{\pi}{16} \times 60 \times d^3$$

$$\Rightarrow d = \sqrt[3]{\frac{5307.86 \times 10^3 \times 16}{\pi \times 60}} = 76.68 mm$$

CHAPTER:6

Q. Write assumptions of pure torsion 2014(w) 2015(w) 7,(b)

Ans: i) The material of shaft is uniform throughout

ii) The twist along the shaft is uniform

iii) Normal cross section of shaft which were plane and circular before twist remain plane and circular after twist.

iv) All diameters and normal cross section which were straight before twist remains straight with their magnitude unchanged after twist.

Q. Define torsion 2014(w), 7(a)

Ans: The product of turning force and the distance between the point of application of force and the axis of shaft is known as torque and shsft is subjected to torsion.

Q. Define fatigue and creep 2014(w), 8(a)

Ans: **Fatigue**: when a material is subjected to repeated stresses it fails at stresses below yield point stress at which failure of material takes place. Such type of failure of material is known as fatigue.

Creep: When a part is subjected to constant stress at high temperature for a longer period of time it will undergo slow and permanent deformation known as creep.

Problem:

What diameter of shaft will be required to transmit 80 kw at 80 rpm. If the maximum torque is 30 percent more than the mean and limit of torsional stress is 56 MPa. 2014(w), 7(c)

Let d = diameter of shaft

Power transmitted, $P = 80\ \text{kw} = 80 \times 10^3\ \text{w}$

Speed of shaft, $N = 80$ rpm

$T_{max} = 1.3 \times T_{mean}$

Allowable shear stress, $\tau = 56\ \text{MPa} = 56\ \text{N/mm}^2$

$$\text{Meantorque}, T_{mean} = \frac{P \times 60}{2\pi N} \text{N} - \text{m}$$

$$= \frac{80 \times 10^3 \times 60}{2 \times \pi \times 80} \text{N} - \text{m} = 9554.14 \text{N} - \text{m}$$

$$= 9554.14 \times 10^3 \text{N} - \text{mm}$$

$$T_{max} = 1.3 \times T_{mean} = 1.3 \times 9554.14 \times 10^3 \text{N} - \text{mm}$$

$$= 12420.38 \times 10^3 \text{N} - \text{mm}$$

$$T_{max} = \frac{\pi}{16} \times \tau \times d^3$$

$$\text{or} (12420.38 \times 10^3) = \frac{\pi}{16} \times 56 \times d^3$$

$$d = \sqrt[3]{\frac{(12420.38) \times 10^3 \times 16}{\pi \times 56}} = 48.36\text{mm}$$

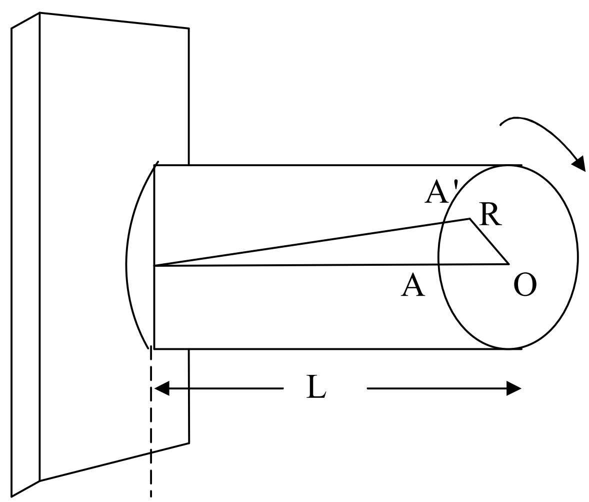
Hence diameter of shaft is 48.36 mm

Q. ***Derive the relation*** $\frac{T}{J} = \frac{\tau}{r} = \frac{c\upsilon}{l}$ ***2003, (8) 2010, 2(g) 2005, 1(c)***

Ans: Consider a circular shaft of 'R' radius subjected to torque 'T' as shown in the figure. Due to this torque OA will deformed to OA' throw an angle θ rad. And CA will deformed to CA'

In Δ CAA'

Tan φ = AA'/L



[When φ is very small, tan φ = φ]

⇒ tan φ = φ = AA'/L ---------------(i)

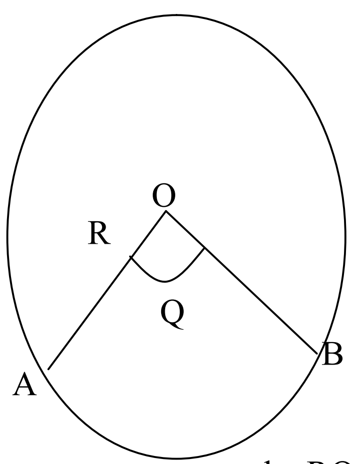
But $\phi \ \frac{\tau}{c} = (\because c = \frac{r}{\phi}) ---(ii)$

$e(i)0 = e(ii)$

$\Rightarrow \quad \frac{\tau}{c} = \frac{RQ}{L}$

$\Rightarrow \frac{\tau}{R} = \frac{CQ}{L}$

Also we know that torque trqansmitted by the shaft



$l \qquad l = RQ$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$\Rightarrow \tau = \frac{16T}{\pi D^3}$$

we know that $\frac{\tau}{R} = \frac{CQ}{L}$

or $$\frac{\frac{16T}{\pi D^3}}{\frac{D}{2}} = \frac{CQ}{L}$$

$$\Rightarrow \frac{32T}{\pi D^4} = \frac{CQ}{L}$$

$$\Rightarrow \frac{T}{\frac{\pi D^4}{32}} = \frac{CQ}{L}$$

But we know that

$$\frac{\tau}{R} = \frac{CQ}{L}$$

$$\therefore \frac{\tau}{R} = \frac{T}{J} = \frac{CQ}{L}$$

$$\Rightarrow \left[\frac{T}{J} = \frac{\tau}{R} = \frac{CQ}{L}\right]$$

or $\frac{T}{\frac{\pi}{32} \times D^4} = \frac{\tau}{\frac{D}{2}}$ or $\frac{32T}{\pi D^4} = \frac{2\tau}{D}$

or $$T = \frac{2\tau}{D} \times \frac{\pi D^4}{32} = \frac{\pi}{16} \times \tau \times D^3$$

CHAPTER 6 & 7

Torison & Column

Q.1 What is torsional rigidity 2012, 7(a)

Ans: It can be defined as the torque required to produce a twist of 1 radian per unit length of the shaft.

Q.2. Define creep & fatigue 2007, 1(v)

Ans: Creep: When a part is subjected to a continous stress at high temperature for a long period of time, it will undergo slow but permanent deformation known as creep.

Fatigue: When ever a member is subjected to a repeated stress, mit will fail below its yield point. This phenomenon is known as fatigue.

Q.3. What is crippling load Buckling 2010, 1(h)

Ans: The load at which the column just buckles is called crippling load.

Q.4. Define slenderness ratio

Ans: The ratio of equivalent length of column to minimum radius of gyration of column is known as slenderness ratio . slenderness ratio = Le/K

Q.5. Define column 2013,1(a) 2006 , 1(vi)

Ans: A column is a fixed long vertical member or bar which is generally subjected to axial compressive load.